Math 1231 Fall 2024
Single-Variable Calculus I Section 11
Mastery Quiz 5
Due Monday, October 6

This week's mastery quiz has three topics. Everyone should submit on S4. If you have a 4/4 on M2 in Blackboard, you don't need to submit it again, and if you have a 2/2 on S3 you don't need to submit that again.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Monday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 2: Computing Derivatives
- Secondary Topic 3: Linear Approximation
- Secondary Topic 4: Rates of Change

Name:

Recitation Section:

Major Topic 2: Computing Derivatives

(a) Compute $\dfrac{d}{dx}\csc^{7/3}\left(\dfrac{x^2+\sin(x)}{\sqrt{x}-\cot(x)}\right)$.

Solution:

$$\frac{7}{3}\csc^{4/3}\left(\frac{x^2+\sin(x)}{\sqrt{x}-\cot(x)}\right)\left(-\csc\left(\frac{x^2+\sin(x)}{\sqrt{x}-\cot(x)}\right)\cot\left(\frac{x^2+\sin(x)}{\sqrt{x}-\cot(x)}\right)\right)$$
$$\cdot\frac{(2x+\cos(x))(\sqrt{x}-\cot(x))-\left(\frac{1}{2}x^{-1/2}+\csc^2(x)\right)(x^2+\sin(x))}{(\sqrt{x}-\cot(x))^2}.$$

(b) Compute $\dfrac{d}{dx}\sec\left(\dfrac{x^3-x}{\sqrt[5]{x}+1}\right)=$

Solution:

$$\sec\left(\frac{x^3-x}{\sqrt[5]{x}+1}\right)\tan\left(\frac{x^3-x}{\sqrt[5]{x}+1}\right)\frac{(3x^2-1)(\sqrt[5]{x}+1)-\frac{1}{5}x^{-4/5}(x^3-x)}{(\sqrt[5]{x}+1)^2}$$

Secondary Topic 3: Linear Approximation

(a) Give a formula for a linear approximation of $f(x)=\sqrt{x^3+1}$ near the point $a=2$. Use your answer to estimate $f(2.1)$.

Solution:

$$\begin{aligned} f'(x) &= \frac{1}{2}(x^3+1)^{-1/2}3x^2 \\ f'(a) &= \frac{1}{2}(9)^{-1/2}\cdot 12 = 2 \\ f(x) &= f(a)+f'(a)(x-a) = 3+2(x-2). \end{aligned}$$

$f(2.1)\approx 3+2(.1)=3.2.$

(b) Write the equation for the tangent line to $g(x)=2x-\tan(x)$ at the point $a=\pi$.

Solution:

$$\begin{aligned} g(\pi) &= 2\pi-0=\pi \\ g'(x) &= 2-\sec^2(x) \\ g'(\pi) &= 2-1=1 \\ y &= 2\pi+(x-\pi) \end{aligned}$$

Secondary Topic 4: Rates of Change

(a) Suppose the distance between two particles in centimeters is given as a function of time in seconds by the formula $d(t) = t^3 + 4t^2 + 5t + 4$.

(i) When is the velocity zero?

(ii) When is the acceleration zero?

Solution:

(i) $d'(t) = 3t^2 + 8t + 5 = (3t + 5)(t + 1)$ so the velocity is zero when $t = -1, -5/3$.

(ii) $d''(t) = 6t + 8$ is zero when $t = -4/3$.

(b) Suppose that a factory produces widgets, and if p people work at the factory then they will produce a total of $W(p) = 30\sqrt{p}$ widgets.

(i) What are the units of $W'(p)$? What does it represent physically? What does it mean if W' is big?

(ii) Calculate $W'(9)$. What does this tell you physically? What physical observation could you make to check your calculation?

Solution:

(i) $W'(p)$ has units of widgets per person. It describes the rate at which the number of widgets increases as we add more people to the factory (this is called the marginal product of labor, but you don't need to know that). If it's large, that means that adding one more person to the factory will let us produce a lot more widgets.

(ii) $W'(p) = \frac{15}{\sqrt{p}}$ so $W'(9) = 5$. So moving from nine people to ten people working at the factory will lead to the production of five extra widgets.