Math 1231 Fall 2024
Single-Variable Calculus I Section 11
Mastery Quiz 6
Due Wednesday, October 9

This week's mastery quiz has three topics. If you have a 4/4 on M2, or a 2/2 on S3 or S4, you don't need to submit them. If you're unsure about your current grade, please check Blackboard.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 2: Computing Derivatives
- Secondary Topic 3: Linear Approximation
- Secondary Topic 4: Rates of Change

Name:

Recitation Section:

Major Topic 2: Computing Derivatives

(a) Compute $\frac{d}{dx}\sec\left(\frac{x^2+1}{\sqrt{x^3-2}}\right) =$

Solution:

$$\sec\left(\frac{x^2+1}{\sqrt{x^3-2}}\right)\tan\left(\frac{x^2+1}{\sqrt{x^3-2}}\right)\frac{2x\sqrt{x^3-2}-(x^2+1)\frac{1}{2}(x^3-2)^{-1/2}\cdot 3x^2}{x^3-2}.$$

(b) Compute $\frac{d}{dt}\sqrt[5]{\frac{\tan^2(t^2+1)+2}{\sin(2t)-2t}}$.

Solution:

$$\frac{d}{dt}\sqrt[5]{\frac{\tan^2(t^2+1)+2}{\sin(2t)-2t}} = \frac{1}{5}\left(\frac{\tan^2(t^2+1)+2}{\sin(2t)-2t}\right)^{-4/5}$$
$$\cdot\frac{\left(2\tan(t^2+1)\sec^2(t^2+1)2t\right)\left(\sin(2t)-2t\right)-\left(2\cos(2t)-2\right)\left(\tan^2(t^2+1)+2\right)}{\left(\sin(2t)-2t\right)^2}.$$

Secondary Topic 3: Linear Approximation

(a) Estimate $\sqrt[4]{15}$ using a linear approximation of the function $\sqrt[4]{x}$ at the point 16.

Solution: We have $h(x)=\sqrt[4]{x}$ and so $h'(x)=\frac{1}{4}x^{-3/4}=\frac{1}{4\sqrt[4]{x^3}}$. Thus in particular, we have $h'(16)=\frac{1}{4\sqrt[4]{16^3}}=\frac{1}{4\cdot 2^3}=1/32$.

The tangent line approximation is

$$y-2=\frac{1}{32}(x-16)$$

so we have

$$f(x)\approx\frac{1}{32}(x-16)+2$$
$$f(15)\approx\frac{1}{32}(-1)+2=2-\frac{1}{32}=\frac{63}{32}.$$

(b) Write the equation for the tangent line to $g(x)=\frac{x+2}{x-5}$ at the point $a=6$.

Solution:

$$g'(x)=\frac{(x-5)-(x+2)}{(x-5)^2}$$
$$g'(6)=\frac{1-8}{1^2}=-7$$
$$y=8-7(x-6)$$

Secondary Topic 4: Rates of change

(a) Suppose the distance between two particles in centimeters is given as a function of time in seconds by the formula $d(t) = t + \frac{1}{t}$.

(i) When is the velocity zero?

(ii) When is the acceleration zero?

Solution:

(i) $d'(t) = 1 - 1/t^2$ so the velocity is zero when $t = \pm 1$.

(ii) $d''(t) = 2/t^3$ is never zero.

(b) The force a magnet exerts on a piece of iron depends on the distance between the magnet and the metal. Let $F(d) = \frac{2}{d^2}$ give the force exerted by the magnet in Newtons, where d is the distance between them in meters.

(i) What are the units of $F'(d)$? What does it $F'(d)$ represent physically? What would it mean if $F'(d)$ is big?

(ii) Calculate $F'(2)$. What does this tell you physically? What physical observation could you make to check your calculation?

Solution:

(i) The derivative is the rate at which the amount of force changes as you change the distance between the magnet and the iron; its units are Netwons per meter. If $F'(d)$ is big, that means that moving the magnet a little bit will change the force on it by a lot.

(ii) $F'(d) = \frac{-4}{d^3}$ so $F'(3) = \frac{-4}{8} = -1/2$. This means that moving the iron another meter away from the magnet should reduce the force by about half a Newton.