

Math 1231: Single-Variable Calculus 1
George Washington University Fall 2024
Recitation 6

Jay Daigle

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Problem 1. Find an equation for the tangent line to $y = 6 \cos x$ at $(\pi/3, 3)$.

Problem 2. Find an equation for the tangent line to $y \cos(x) = 1 + \sin(xy)$ at the point $(0, 1)$.

Problem 3. Suppose we have some function f such that $8f(x) + x^2(f(x))^3 = 24$, and we know that $f(4) = 1$. (Say we've measured this experimentally and now want to understand or compute with the function). Now suppose we want to estimate $f(5)$ (without having to solve the equation).

- (a) Use implicit differentiation on the equation $8f(x) + x^2(f(x))^3 = 24$ to find a formula relating x , $f(x)$ and $f'(x)$.
- (b) Use this formula to determine $f'(4)$.
- (c) Find a formula for the linear approximation of f near 4.
- (d) Estimate $f(5)$.

Problem 4 (Bonus). (a) If $\sqrt{xy} = x^2y - 2$, find a formula for $\frac{dy}{dx}$ in terms of x and y .

- (b) Find an equation of the tangent line at the point $(1, 4)$.

Problem 5 (Bonus). A rectangle is getting longer by one inch per second and wider by two inches per second. When the rectangle is 5 inches long and 7 inches wide, how quickly is the area increasing?

- (a) Draw a picture of this situation.
- (b) What is the question you're trying to answer? What do you expect it to look like? Should it be positive or negative? What units do you expect?
- (c) What equation should we use here, and why?
- (d) Use a derivative to calculate the answer to the question. Does your answer make sense?
- (e) To check things: how long and wide will the rectangle be after one inch? How much will the area have increased? Does that make sense with your answer to the related rates problem?
- (f) Bonus: where have we seen basically this argument before?