

Math 1231 Fall 2024  
Single-Variable Calculus I Section 11  
Mastery Quiz 7  
Due Monday, October 14

This week's mastery quiz has four topics. Everyone should submit on S5 and S6. If you have a 4/4 on M2 in Blackboard, you don't need to submit it again, and if you have a 2/2 on S4 you don't need to submit that again. (Note the midterm can improve your mastery scores, so do try to check Blackboard! We'll get the grades up when we can.)

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Monday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

**Topics on This Quiz**

- Major Topic 2: Computing Derivatives
- Secondary Topic 4: Rates of Change
- Secondary Topic 5: Implicit Differentiation
- Secondary Topic 6: Related Rates

**Name:**

**Recitation Section:**

## Major Topic 2: Computing Derivatives

(a) Compute  $\frac{d}{dx} \cos(\sec^2(x^3 \tan(x^2)))$ .

(b) Compute  $\frac{d}{dx} \left( \frac{x \csc(x)}{\sqrt{x^3 - x}} \right)^3$

## Secondary Topic 4: Rates of Change

(a) Let  $F(x) = \frac{1}{x} + 1$  be the amount of pressure exerted on a beam in pounds per square inch at a point  $x$  inches to the right of its left end.

(i) What are the units of  $F'(x)$ ? What does  $F'(x)$  represent physically? What would it mean if  $F'(x)$  is big?

(ii) Compute  $F'(5)$ . What does this tell you physically? What physical observation could you make to check your calculation?

(b) Suppose the height of a particle in centimeters is given as a function of time in seconds  $p(t) = t^3 - 3t$ .

(i) When is the velocity zero?

(ii) When is the acceleration zero?

## Secondary Topic 5: Implicit Differentiation

(a) Find a formula for  $\frac{d^2y}{dx^2}$  if  $x^3 = xy + 1$ .

(b) Find an equation for the line tangent to the curve  $x^2y - xy^3 = xy + 3$  at the point  $(3, 1)$ .

## Secondary Topic 6: Related Rates

(a) A snowball is melting such that its surface area is decreasing at  $1\text{cm}^2/\text{min}$ . When the radius is  $8\text{cm}$ , how quickly is the radius decreasing?

(The surface area of a sphere of radius  $r$  is  $4\pi r^2$ .)

(a) Choose an equation to use for this problem, and explain why you chose that equation.

(b) Use calculus to answer the question. Make sure you answer with a complete sentence.

- (b) A balloon is rising at a constant speed of 5 feet per second. A boy is cycling along a straight road at a speed of 15 feet per second. When he passes under the balloon, it is 45 feet above him. We want to know how fast is the distance between the boy and the balloon is increasing 3 seconds later.
- (a) Choose an equation to use for this problem, and explain why you chose that equation.
- (b) Use calculus to answer the question. Make sure you answer with a complete sentence.

