Math 1231 Fall 2024
Single-Variable Calculus I Section 11
Mastery Quiz 7
Due Wednesday, October 16

This week's mastery quiz has three topics. Everyone should submit on S5 and S6. If you have a 4/4 on M2, you don't need to submit it. If you're unsure about your current grade, please check Blackboard.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 2: Computing Derivatives
- Secondary Topic 5: Implicit Differentiation
- Secondary Topic 6: Related Rates

Name:

Recitation Section:

Major Topic 2: Computing Derivatives

(a) Compute $\dfrac{d}{dx}\dfrac{\sin(\sec(x^2+1))}{x^4+\cos(x)} =$

Solution:

$$\frac{\big(\cos(\sec(x^2+1))(\sec(x^2+1)\tan(x^2+1))2x\big)(x^4+\cos(x)) - (4x^3-\sin(x))\sin(\sec(x^2+1))}{(x^4+\cos(x))^2}.$$

(b) Compute $\dfrac{d}{dx}\tan(x^2+\csc^2(x^3+\sin(x^5)))$.

Solution:

$$\frac{d}{dx}\tan(x^2+\csc(x^3+\sin(x^5)))$$
$$= \sec^2(x^2+\csc(x^3+\sin(x^5)))\Big(2x - 2\csc^2(x^3+\sin(x^5))\cot(x^3+\sin(x^5))\Big)\cdot(3x^2+\cos(x^5)\cdot 5x^4).$$

Secondary Topic 5: Implicit Differentiation

(a) Find a formula for y' in terms of x and y if $\sqrt{x+y} = x^3y^2$.

Solution:

$$\begin{aligned}
\frac{1}{2}(x+y)^{-1/2}(1+y') &= 3x^2y^2 + 2x^3yy' \\
\frac{1}{2}(x+y)^{-1/2}y' - 2x^3yy' &= 3x^2y^2 - \frac{1}{2}(x+y)^{-1/2} \\
y' &= \frac{3x^2y^2 - \frac{1}{2}(x+y)^{-1/2}}{\frac{1}{2}(x+y)^{-1/2} - 2x^3y}.
\end{aligned}$$

(b) Write a tangent line to the curve $x^2y^2 = 5 + x + y$ at the point $(1,3)$.

Solution: Implicit differentiation gives us

$$\begin{aligned}
2xy^2 + 2x^2yy' &= 1 + y' \\
2\cdot 1\cdot 9 + 2\cdot 1^2\cdot 3\cdot y' &= 1 + y' \\
18 + 6y' &= 1 + y' \\
5y' &= -17 \\
y' &= -17/5
\end{aligned}$$

and thus the tangent line has equation

$$y - 3 = \frac{-17}{5}(x - 1).$$

Alternatively we can compute

$$\begin{aligned} 2xy^2 + 2x^2yy' &= 1 + y' \\ y'(2x^2y - 1) &= 1 - 2xy^2 \\ y' &= \frac{1 - 2xy^2}{2x^2y - 1} \end{aligned}$$

Secondary Topic 6: Related Rates

(a) The surface area of a cube is given by the formula $A = 6s^2$ where s is the length of a side. If the side lengths are increasing by 2 inches per second, how fast is the surface area increasing when the area is 54 square inches?

Solution: We have the data $A = 6s^2, A = 54, s' = 2$. We take a derivative and see that $A' = 12ss'$, so we need to find s. But when $A = 54$ we have

$$\begin{aligned} 54 &= 6s^2 \\ 9 &= s^2 \\ 3 &= s \end{aligned}$$

and thus

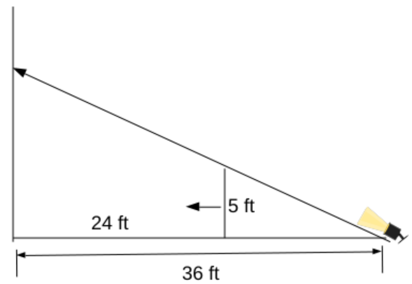
$$A' = 12ss' = 12 \cdot 3 \cdot 2 = 72$$

so the area is increasing at 72 square inches per second.

(b) A spot light is on the ground 36 ft away from a wall and a 5 ft tall person is walking towards the wall at a rate of 4 ft/sec. How fast is the height of the shadow changing when the person is 24 feet from the wall? Is the shadow increasing or decreasing in height at this time?

  (a) Choose an equation to use for this problem, and explain why you chose that equation.

  (b) Use calculus to answer the question. Make sure you answer with a complete sentence that clearly and directly answers the question.



Solution: Let h be the height of the shadow, and d be the distance between the wall and the person. Then we want to find h'. We currently have $d = 24$. We know by similar triangles that $\frac{36-d}{36} = \frac{5}{h}$, which tells us that currently $h = 15$.

Then we have $d' = -4$. We compute

$$\begin{aligned} \frac{-d'}{36} &= \frac{-5h'}{h^2} \\ \frac{1}{9} &= \frac{-h'}{45} \\ h' &= \frac{-45}{9} = -5. \end{aligned}$$

Thus the shadow's height is decreasing by 5 feet per second.