Math 1231: Single-Variable Calculus 1

George Washington University   Fall 2024

Recitation 9

Jay Daigle

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Problem 1. Let $g(x) = x \tan(x)$. We want to sketch a graph of g.

1. What is the domain of g?

2. For simplicity, let's just look at $[-\pi/2, \pi/2]$. What can you say about any asymptotes it has?

3. Does this function have any roots you can find?

4. $g'(x) = \frac{\sin(x)\cos(x) + x}{\cos^2(x)}$. What are the critical points?

   (Hint: when is $\sin(x)\cos(x)$ positive and when is it negative?)

5. What are the critical values?

6. Where is g increasing and decreasing? Does it have maxima or minima?

7. $g''(x) = 2\sec^2(x)(1 + x\tan(x))$. Where are the potential points of inflection, and what are their values? Where is h concave up and down?

8. Sketch the graph.

Problem 2. Let $h(x) = \frac{x+2}{x-1}$. We want to sketch a graph of h.

1. What is the domain of h? What can you say about any asymptotes it has?

2. Does this function have any roots? Where?

3. What happens as x approaches $+\infty$? $-\infty$?

4. $h'(x) = -3(x-1)^{-2}$. What are the critical points and values?

5. Where is h increasing and decreasing? Does it have maxima or minima?

6. $h''(x) = 6(x-1)^{-3}$. Where are the potential points of inflection? Where is h concave up and down?

7. Sketch the graph.

Problem 3 (Bonus). Let $g(x) = x^5 - 4x^3 + 4x + 7$. We want to sketch a graph of g.

1. What is the domain of g? What can you say about any asymptotes it has?

2. Does this function have any roots you can find?

3. What happens as x approaches $+\infty$? $-\infty$?

4. $g'(x) = 5x^4 - 12x^2 + 4$. What are the critical points?

   (Hint: if we set $u = x^2$ this becomes a quadratic, and we can factor it.)

5. What are the critical values?

   (Hint: $g(x) = 7 + x(x^4 - 4x^2 + 4) = 7 + x(u^2 - 4u + 4)$.)

6. Where is g increasing and decreasing? Does it have maxima or minima?

7. $g''(x) = 20x^3 - 24x$. Where are the potential points of inflection, and what are their values? Where is h concave up and down?

8. Sketch the graph.