

# Math 1231 Practice Midterm Solutions

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- These are the instructions you will see on the real test, next week. I include them here so you know what to expect.
- You will have 75 minutes for this test.
- You are not allowed to consult books or notes during the test, but you may use a one-page, one-sided, handwritten cheat sheet you have made for yourself ahead of time.
- You may not use a calculator.
- This test has eight questions, over five pages. **You should not answer all eight questions.**
  - The first two problems are two pages, representing topics M1 and M2. You should do both of them, and they are worth 30 points each.
  - The remaining six problems represent topics S1 through S6. You will be graded on your best three, with a few possible bonus points if you also do well on the others.
  - Doing three secondary topics well is much better than doing five or six poorly.
  - If you perform well on a question on this test it will update your mastery scores. Achieving a 27/30 on a major topic or 9/10 on a secondary topic will count as getting a 2 on a mastery quiz.

**Problem 1** (M1). Compute the following using methods we have learned in class. Show enough work to justify your answers.

(a) Find the tangent line to  $h(x) = \arcsin(e^x)$  at  $\ln(1/2)$ .

(b)  $\int_1^2 \frac{e^{1/x}}{x^2} dx =$

(c)  $\int \frac{\cos(x) \sin(x)}{1 + \cos^4(x)} dx =$

**Problem 2** (M2). Compute the following integrals using methods we have learned in class. Show enough work to justify your answers.

(a)  $\int \frac{2x + 1}{\sqrt{x^2 - 1}} dx$

(b)  $\int x \sec^2 x dx$

(c)  $\int_0^1 \frac{3x^2 - 6x + 1}{(x^2 - x - 1)(x - 2)} dx$

**Problem 3** (S1). Let  $f(x) = \sqrt[3]{x^5 + x^4 + x^3 + x^2 + 2x}$ . Find  $(f^{-1})'(4)$ .

**Problem 4** (S2). Find  $\lim_{x \rightarrow 0} \frac{2 \sin(x) - \sin(2x)}{x - \sin(x)}$ .

**Problem 5** (S3). Use Simpson's rule and six intervals to estimate  $\int_0^6 x^4 dx$ . Give an upper bound for the error on this approximation.

**Problem 6** (S4). Compute  $\int_1^{10} \frac{1}{\sqrt[3]{x-2}} dx$ .

**Problem 7** (S5). Find the surface area of the surface obtained by rotating  $y = \sqrt{5 + 4x}$  for  $-1 \leq x \leq 1$  about the  $x$ -axis.

**Problem 8** (S6). Find a (specific) solution to the initial value problem  $y'/x - y = 1$  if  $y(0) = 3$