Math 1232: Single-Variable Calculus 2
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Recitation 0

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What is a *function*?

Sometimes we have a function and we want to undo it. That is, we want to know the output and find the input. This is a basically reasonable question: "What do I have to do if I want to get a specific result" is something we find ourselves thinking a lot. But now we want to look at the mathematical side of it.

Definition 0.1. If f is a function and $(g \circ f)(x) = x$ for every x in the domain of f, then we say g is an *inverse* of f.

Example 0.2.
- If $f(x) = x$ then $g(y) = y$ is an inverse to f.
- If $f(x) = 5x + 3$ then $g(y) = (y - 3)/5$ is an inverse to f.

Problem 1. (a) Can you find an inverse for $f(x) = x^2$?

(b) Can you find an inverse for $f(x) = x^3$?

(c) What makes these two functions different?

(d) What needs to happen for us to be able to invert or "undo" a function?

(e) What would this look like on a graph? [Hint: the answer is not the vertical line test, but thinking about the vertical line test will help you.]

Definition 0.3. A function f is *1-1* or *one-to-one* (or *injective*) if, whenever $f(a) = f(b)$, we know that $a = b$.

Any invertible function has to be one-to-one. Less obviously, any one-to-one function is invertible.

Problem 2. (a) Is the function $f(x) = |x|$ one-to-one? Prove it is, or find a counterexample.

(b) Is the function $g(x) = 5x^3 + 3$ one-to-one? Prove it is, or find a counterexample.

(c) Find an inverses for any of these functions that were one-to-one.

Problem 3. Consider the function $f(x) = x^4$.

(a) Is this one-to-one? Is it invertible?

(b) Then what is $\sqrt[4]{x}$?

(c) What needs to happen for $\sqrt[4]{x}$ to be an inverse?

(d) Can you find a completely different set of numbers where f is invertible?? Find an inverse on that domain.

Problem 4. Let $f(x) = x^5 + x$.

(a) Is this function one-to-one? You won't be able to prove it directly from the definition, but you can use calculus to make a clear argument.

(b) Can you find an inverse for this function?

(c) Can you find $f^{-1}(2)$? $f^{-1}(34)$? $f^{-1}(-2)$?

(d) Can you find $(f^{-1})'(2)$?

(e) Can you find $(f^{-1})'(34)$? $(f^{-1})'(-2)$?