Math 1232: Single-Variable Calculus 2
George Washington University Fall 2024
Recitation 1

Jay Daigle

August 30, 2024

Problem 1. Consider $f(x) = \cos(x)$.

(a) Is this function one-to-one? Why or why not?

(b) What domains can you restrict it to to get a one-to-one function?

(c) What value "should" you pick to solve $\cos(x) = 0$? What about $\cos(x) = 1$? $\cos(x) = -1$?

(d) What domain should you pick to create an inverse?

Problem 2. In class we talked about the function $f(x) = x^5 + x$. This function is 1-1, but we can't write down an inverse for it. But there are still some things we can say about the inverse.

(a) Can you find $f^{-1}(2)$? $f^{-1}(34)$? $f^{-1}(-2)$?

(b) Can you find $(f^{-1})'(2)$?

(c) Can you find $(f^{-1})'(34)$? $(f^{-1})'(-2)$?

Problem 3. Let $g(x) = \sqrt[3]{x^3 + x + 6}$.

(a) Can you compute an inverse for g?

(b) Can you find $\left(g^{-1}\right)'(2)$?

(a) The function is invertible, since it's increasing. You even, in theory, could find the inverse. But realistically you're not going to; the formula is:

$$g^{-1}(y) = \frac{\sqrt[3]{\frac{2}{3}}}{\sqrt[3]{-9y^3 + \sqrt{3}\sqrt{27y^6 - 324y^3 + 976} + 54}} - \frac{\sqrt[3]{-9y^3 + \sqrt{3}\sqrt{27y^6 - 324y^3 + 976} + 54}}{\sqrt[3]{2} \cdot 3^{2/3}},$$

and I wouldn't expect anyone to successfully find or work with that.

(b) This is much easier. By the Inverse Function Theorem we know that

$$\begin{aligned} (g^{-1})'(x) &= \frac{1}{g'(g^{-1}(x))} \\ g'(x) &= \frac{1}{3}(x^3 + x + 6)^{-2/3}(3x^2 + 1). \end{aligned}$$

We just need to find $g^{-1}(2)$, which we can, essentially, solve by guessing and checking: and it turns out that $g(1) = 2$, so $g^{-1}(2) = 1$. So we have

$$\begin{aligned} g'(1) &= \frac{1}{3}(1^3 + 1 + 6)^{-2/3}(3(1)^2 + 1) = \frac{1}{3}8^{-2/3}(4) = \frac{1}{3} \\ (g^{-1})'(x) &= \frac{1}{g'(g^{-1}(2))} = \frac{1}{g'(1)} = 3. \end{aligned}$$

Problem 4. (a) Consider the functions $f(x) = x^3 - x^2 + x$ and $g(x) = x^3 - x^2 - x$. Which one is invertible and why?

(b) Consider the functions $f(x) = 3^x + x$ and $g(x) = 3^x - x$. Can you figure out which one is invertible?

Problem 5. (a) Compute $\log_3(6) + \log_3(9/2)$.

(b) Compute $\log_4(8) - \log_4(2)$.

(c) Rewrite the expression $\log_5(15) + \log_5(75) - \log_5(12)$ as an integer plus a logarithm.

(d) Solve $e^{5-3s} = 10$.