Math 1232: Single-Variable Calculus 2
George Washington University   Fall 2024
Recitation 10

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Problem 1. Analyze the convergence of the following series. Write clean arguments that establish whether they diverge, converge conditionally, or converge absolutely. Think about what tools/tests you want to use, and why.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n!}{2^n}$

(b) $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^{n-1}}{\ln(n)}$

Problem 2 (Bessel Function)**.** The Bessel function (of order 0) is critical to any physics done in cylindrical coordinates, and thus any physics that occurs on a cylinder. We saw it earlier as the solution to the differential equation $x^2y'' + xy' + x^2y = 0$, but it can also be given by the power series:

$$J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}.$$

What is the radius of convergence? What is the interval of convergence?

Problem 3. What is the interval of convergence of

$$\sum_{n=0}^{\infty} \frac{n^2(x-1)^n}{7^{n+2}}?$$

Problem 4 (Bonus)**.** In class we showed that $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ converges to a number between 1 and 1/2. I claimed that it converged to $\ln 2$. But this convergence is conditional, and that matters.

(a) Write out the first twelve terms of this series.

(b) Reorganize them so that you have the same collection of numbers add one and then subtract two, then add one, then subtract two, and so on. (You'll have some extras left over and that's fine; remember you have an infinite list of terms.)

(c) What does each triplet look like? Can you simplify that somehow so it looks like something we recognize? (Hint: what happens if you combine the first two terms of a triplet?)

(d) Can you figure out what this sequence of partial sums converges to?