Math 1232 Fall 2024
Single-Variable Calculus 2 Section 11
Mastery Quiz 2
Due Monday, September 9

This week's mastery quiz has one topic. If you already have a 2/2 on S1 (as shown on Blackboard), you don't need to submit it. If you have a 1/2 or 0/2, you should submit this topic again.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Monday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Secondary Topic 1: Invertible Functions

Name:

Recitation Section:

S1: Invertible Functions

(a) Find a formula for the inverse of $g(x) = (x - 1)^3 + 3$.

   Solution:

$$y = (x - 1)^3 + 3y - 3 \qquad\qquad = (x - 1)^3$$
$$\sqrt[3]{y - 3} = x - 1$$
$$x = 1 + \sqrt[3]{y - 3}$$

   so $g^{-1}(y) = 1 + \sqrt[3]{y - 3}$. (You can use whichever variable you like in your formula.)

(b) Showing your work, compute $\log_3(90) + \log_3(3/2) - \log_3(5)$.

   Solution:

$$\log_3(90) + \log_3(3/2) - \log_3(5) = \log_3(90 \cdot 3/2/5) = \log_3(27) = 3.$$

(c) Let $h(x) = x^7 + 3x^3 + 1$. Compute $(h^{-1})'(5)$.

   Solution: By the Inverse Function Theorem, we know that

$$(h^{-1})'(5) = \frac{1}{h'(h^{-1}(5))}.$$

   Guess and check shows that $h(1) = 5$ so $h^{-1}(5) = 1$. And we know that

$$h'(x) = 7x^6 + 9x^2$$

   and thus
$$h'(1) = 7 + 9 = 16$$

   Thus
$$(h^{-1})'(5) = \frac{1}{16}.$$