Math 1232: Single-Variable Calculus 2
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Recitation 2

Jay Daigle

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Problem 1. Let $h(x) = \ln|x|$. We're going to compute the derivative of this two ways.

(a) If we assume $x > 0$, we can simplify $\ln|x|$. What does it simplify to? What is the derivative?

(b) If $x < 0$, we can also simplify $\ln|x|$. What does it simplify to? This is a little less obvious. What is the derivative?

(c) What about when $x = 0$?

(d) What pattern do we get here?

(e) Now let's approach this a totally different way. Verify that we can define $|x| = \sqrt{x^2}$. How does that work?

(f) Use the chain rule to compute $\ln(\sqrt{x^2})$. Does this match your previous answer?

Solution:

(a) If $x > 0$ then $\ln|x| = \ln(x)$. Then $h'(x) = \frac{d}{dx}\ln(x) = \frac{1}{x}$.

(b) If $x < 0$ then $\ln|x| = \ln(-x)$. Then

$$h'(x) = \frac{d}{dx}\ln(-x) = \frac{1}{-x}\cdot(-1) = \frac{1}{x}.$$

(c) $\ln(x)$ isn't defined for $x = 0$, so neither is $h(x)$.

(d) For $x > 0$ and also for $x < 0$ we get that $h'(x) = \frac{1}{x}$. So we can just use that as a universal derivative rule.

(e) We know that x^2 is always positive, and $\sqrt{x^2}$ will give the positive square root. So if $x > 0$ then $\sqrt{x^2} = x$; but if $x < 0$ then $\sqrt{x^2} = -x$ which is the same as $|x|$.

(f)

$$\begin{aligned} h'(x) &= \frac{1}{\sqrt{x^2}} \cdot \frac{1}{2}(x^2)^{-1/2} \cdot 2x \\ &= \frac{1}{\sqrt{x^2}} \cdot \frac{1}{2\sqrt{x^2}} \cdot 2x \\ &= \frac{1}{2x^2} \cdot 2x = \frac{1}{x}. \end{aligned}$$

Problem 2. Compute the derivative of $(x+1)^{\sqrt{x}}$.

Solution: We can't do this from the derivative rules we already have. But we can use logarithms!

$$\begin{aligned} y &= (x+1)^{\sqrt{x}} \\ \ln|y| &= \ln\left|(x+1)^{\sqrt{x}}\right| = \sqrt{x}\ln|x+1| \\ \frac{y'}{y} &= \frac{1}{2\sqrt{x}}\ln|x+1| + \frac{\sqrt{x}}{|x+1|} \\ y' &= y\left(\frac{1}{2\sqrt{x}}\ln|x+1| + \frac{\sqrt{x}}{|x+1|}\right) \\ &= (x+1)^{\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\ln|x+1| + \frac{\sqrt{x}}{|x+1|}\right). \end{aligned}$$

Problem 3. Use logarithmic differentiation to compute $\frac{d}{dx}\frac{x^3\sqrt{x^2-5}}{(x+4)^3}$.

Solution:

$$\begin{aligned}
y &= \frac{x^3\sqrt{x-5}}{(x+4)^3} \\
\ln|y| &= \ln\left|\frac{x^3\sqrt{x^2-5}}{(x+4)^3}\right| \\
&= 3\ln|x| + \frac{1}{2}\ln|x^2-5| - 3\ln|x+4| \\
\frac{y'}{y} &= \frac{3}{x} + \frac{1}{2}\frac{2x}{x^2-5} - \frac{3}{x+4} \\
y' &= y\left(\frac{3}{x} + \frac{1}{2}\frac{2x}{x^2-5} - \frac{3}{x+4}\right) \\
&= \frac{x^3\sqrt{x^2-5}}{(x+4)^3}\left(\frac{3}{x} + \frac{x}{x^2-5} - \frac{3}{x+4}\right).
\end{aligned}$$

If we want we can even simplify this to

$$y' = \frac{3x^2\sqrt{x^2-5}}{(x+4)^3} + \frac{x^4}{(x+4)^3\sqrt{x^2-5}} - \frac{3x^3\sqrt{x^2-5}}{(x+4)^4}$$

Problem 4. Try to compute the following integrals.

(a) $\int_0^3 e^x\,dx$

(b) $\int_0^{\ln(3)} e^x\,dx$

(c) $\int e^{3x}\,dx$. (Hint: remember u-substitution!)

(d) $\int 3^x\,dx$. Hint: there are a couple ways you can approach this.

Solution:

(a) $\int_0^3 e^x\,dx = e^x|_0^3 = e^3 - 1$.

(b) $\int_0^{\ln(3)} e^x\,dx = e^x|_0^{\ln(3)} = 3 - 1 = 2$.

(c) Let's compute $\int e^{3x}\,dx$. We can take $u = 3x$ so $dx = du/3$, and we have

$$\int e^{3x}\,dx = \int e^u \frac{du}{3} = \frac{1}{3}e^u + C = \frac{1}{3}e^{3x} + C.$$

(d) We can approach $\int 3^x\,dx$ in a couple of different ways. One approach is to think about the rule that $\frac{d}{dx}3^x = 3^x\ln(3)$, and thus $\int 3^x\,dx = \frac{3^x}{\ln(3)} + C$.

The other is to do some algebraic "preprocessing". We know that

$$3^x = (e^{\ln(3)})^x = e^{x\ln(3)}.$$

Thus we're trying to compute

$$\int e^{x\ln(3)}\, dx = \frac{1}{\ln(3)} e^{x\ln(3)} + C = \frac{1}{\ln(3)} 3^x + C.$$