Math 1232 Fall 2024
Single-Variable Calculus 2 Section 11
Mastery Quiz 4
Due Monday, September 23

This week's mastery quiz has two topics. Everyone should submit both of them. Even if you got a 2 on M1 last week, your best two scores count, so you should submit it again.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Monday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 1: Calculus of Transcendental Functions
- Major Topic 2: Advanced Integration Techniques
- Secondary Topic 2: L'Hospital's Rule

Name:

Recitation Section:

M1: Calculus of Transcendental Functions

(a) Compute $\dfrac{d}{dx}x^{\ln(x)}$.

Solution: The simplest approach is to use logarithmic differentiation.

$$\begin{aligned} y &= x^{\ln(x)} \\ \ln(y) &= \ln(x)\ln(x) = \ln(x)^2 \\ \frac{y'}{y} &= 2\ln(x)\frac{1}{x} \\ y &= \frac{2\ln(x)}{x}y = \frac{2\ln(x)x^{\ln(x)}}{x}. \end{aligned}$$

Alternatively, we could compute

$$\begin{aligned} \frac{d}{dx}x^{\ln(x)} &= \frac{d}{dx}\left(e^{\ln(x)}\right)^{\ln(x)} = \frac{d}{dx}e^{\ln(x)^2} \\ &= e^{\ln(x)^2}\cdot 2\ln(x)\frac{1}{x} = \frac{2\ln(x)x^{\ln(x)}}{x}. \end{aligned}$$

(b) Compute $\dfrac{d}{dt}2^t\log_5(t)$.

Solution:

$$\frac{d}{dt}2^t\log_5(t) = 2^t\ln(2)\log_5(t) + 2^t\frac{1}{t\ln(5)}.$$

(c) (Note this is a definite integral)

$$\int_0^2 \frac{e^x}{e^x+1}\,dx =$$

Solution: We can take $u = e^x$ so $du = e^x\,dx$ and

$$\int_0^2 \frac{e^x}{e^x+1}\,dx = \int_1^{e^2}\frac{1}{u+1}\,du = \ln|u+1|\Big|_1^{e^2} = \ln(e^2+1) - \ln(2).$$

S2: L'Hospital's Rule

(a) $\displaystyle\lim_{x\to 0}\frac{x^3-x^2}{x+\sin(x)} =$

Solution: The limits of the top and bottom are both zero, so we can use L'Hospital's Rule:

$$\lim_{x\to 0} \frac{x^3 - x^2 \nearrow^0}{x + \sin(x)_{\searrow 0}} =^{\text{L'H}} \lim_{x\to 0} \frac{3x^2 - 2x \nearrow^0}{1 + \cos(x)_{\searrow 2}} = \frac{0}{2} = 0.$$

Note we *cannot* use L'Hospital's rule a second time, because we don't have an indeterminate form.

(b) $\displaystyle \lim_{x\to 0} \left(\frac{e^x + 1}{2}\right)^{1/x} =$

Solution:

$$\ln y = \frac{1}{x} \ln\left(\frac{e^x+1}{2}\right)$$

$$\lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{\ln\left(\frac{e^x+1}{2}\right)\nearrow^0}{x_{\searrow 0}}$$

$$=^{\text{L'H}} \lim_{x\to 0} \frac{2}{e^x+1} \cdot \frac{e^x}{2} = \lim_{x\to 0} \frac{e^x}{e^x+1} = 1/2$$

$$\lim_{x\to 0} y = e^{1/2}.$$

(c) $\displaystyle \lim_{x\to\infty} \frac{1 + \ln(x) + (\ln(x))^2}{\sqrt{x}} =$

Solution: The top and bottom both approach infinity, so we can use L'Hospital's Rule.

$$\lim_{x\to\infty} \frac{1 + \ln(x) + (\ln(x))^2 \nearrow^\infty}{\sqrt{x}_{\searrow\infty}} =^{\text{L'H}} \lim_{x\to\infty} \frac{1/x + 2\ln(x)/x}{1/(2\sqrt{x})}$$

$$= \lim_{x\to\infty} \frac{2\sqrt{x} + 4\sqrt{x}\ln(x)\nearrow^\infty}{x_{\searrow\infty}}$$

$$=^{\text{L'H}} \lim_{x\to\infty} \frac{2\nearrow^2}{\sqrt{x}_{\searrow\infty}} + \frac{4\ln(x)\nearrow^\infty}{\sqrt{x}_{\searrow\infty}}$$

$$=^{\text{L'H}} 0 + \lim_{x\to\infty} \frac{4/x}{1/2\sqrt{x}} = \lim_{x\to\infty} \frac{8\nearrow^8}{\sqrt{x}_{\searrow\infty}} = 0.$$