

Math 1232: Single-Variable Calculus 2
George Washington University Fall 2024
Recitation 5

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Problem 1. Compute $\int \sin^6(x) dx$.

Problem 2. Compute $\int \sec^6(x) \tan^5(x) dx$ with two different approaches. Do you get the same answer either way?

Problem 3 (Bonus). Do one of the following two integrals. Explain why you don't want to do the other one.

(a) $\int \tan^2(x) \sec^3(x) dx$

(b) $\int \tan^3(x) \sec^3(x) dx$.

Problem 4. Consider the integral $\int \frac{dx}{\sqrt{4x^2 - 1}}$.

(a) Which trig function would let us simplify that square root, and what identity are we using?

(b) What trigonometric substitution should we use here?

(c) Compute the antiderivative.

(d) Make sure to substitute your x back into the equation!

Problem 5. We want to find $\int \frac{x^5 + x - 1}{x^3 + 1} dx$.

(a) What's the first tool we need to apply here? (Hint: not partial fractions!)

- (b) Once we get it in a more manageable form, things should simplify out nicely. What is the final integral?

Problem 6. We've looked briefly at the integral $\int \frac{1}{1+e^x} dx$. Let's try it again with our new tools.

- (a) Try the substitution $u = e^x$. What do you get? What tools can apply to the result?
- (b) Do a partial fractions decomposition to get the integral.

Problem 7 (Bonus). Let's see if we can work out the integral of secant. This isn't at all obvious!

- (a) We want $\int \sec(x) dx = \int \frac{1}{\cos(x)} dx$. Since this is a fraction, we can multiply the top and bottom through by $\cos(x)$. This makes the expression more complicated, but it does allow us to use a trig identity. What do we get?
- (b) Now we can do a u substitution. What u substitution seems reasonable? Does it help us at all?
- (c) Now we can use partial fractions to finish the problem off. We wind up with an awkward answer, but an answer.
- (d) The most common formula for the integral of $\sec(x)$ is $\ln|\sec(x) + \tan(x)| + C$. Is that the same as what you got? (Hint: use logarithm laws and multiplication by the conjugate.)

Problem 8 (Bonus). What if we want to find $\int \frac{x^4 + 6x^3 + 4x^2 + 8x + 11}{(x-1)^2(2x+1)(x^2+4x+5)} dx$?