

Math 1232: Single-Variable Calculus 2  
George Washington University Spring 2023  
Recitation 6

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**Problem 1.** We want to find the cross-sectional area of a two-meter-long airplane wing. We measure its width every 20 centimeters, and get: 5.8, 20.3, 26.7, 29.0, 27.6, 27.3, 23.8, 20.5, 15.1, 8.7, 2.8. Use the trapezoidal rule and Simpson's rule to estimate the area of the wing.

**Problem 2.** Consider the function  $f(x) = x^2 + 1$ .

- (a) Use the trapezoid rule with six intervals to estimate  $\int_{-4}^2 f(x) dx$ .
- (b) Use the midpoint rule with six intervals to estimate  $\int_{-4}^2 f(x) dx$ .
- (c) Use Simpson's rule with six intervals to estimate  $\int_{-4}^2 f(x) dx$ .
- (d) Which of these do you expect to be most accurate? Which do you expect to be least accurate?
- (e) Compute  $\int_{-4}^2 f(x) dx$ . What do you find? Why?

**Problem 3.** Let  $g(x) = e^{-x^2}$ , and suppose we want to compute  $\int_{-1}^2 e^{-x^2} dx$ , and get the answer correct to two decimal places.

- (a) We can compute that  $g''(x)$  varies between  $-2$  and  $.9$  when  $x$  is in  $[-1, 2]$ . What value should we take for  $K$ ?
- (b) How many subintervals should we use to get the answer correct to within two decimal places using the trapezoid rule?

- (c) How many subintervals should we use to get the answer correct to within two decimal places using the midpoint rule?
- (d) We can compute that  $g'''(x)$  varies between  $-8$  and  $12$ . What value should we take for  $L$ ?
- (e) How many subintervals should we use to get the answer correct to within two decimal places using Simpson's rule?

**Problem 4.** We want to compute  $\int_0^2 \frac{1}{\sqrt[3]{x-1}} dx$ .

- (a) Can you compute an antiderivative? Can you evaluate it at 0 and 2?
- (b) Did part (a) finish the problem? Sketch a picture of the graph. What should we be concerned about?
- (c) Carefully set up a computation that will find  $\int_0^2 \frac{1}{\sqrt[3]{x-1}} dx$ . (Hint: It should have two limit operations in it.)
- (d) What did we learn from this that we didn't learn from (a)?

**Problem 5 (Bonus).** We want to figure out if  $\int_0^{+\infty} e^{-x^2} dx$  converges—that is, if it's finite or infinite.

- (a) If we can find an antiderivative, we can just compute the improper integral directly. Why doesn't that work?
- (b) Since we can't integrate this directly we might want to use a comparison test. We need to find an easy-to-integrate function that's larger than  $e^{-x^2}$ . Find a function  $f(x)$  that makes  $f(x)e^{-x^2}$  easy to integrate.
- (c) If  $f(x) \geq 1$ , then we can just integrate  $f(x)e^{-x^2}$ . Is it?
- (d) This is where we can pull in a trick. Is there some  $a$  where  $f(x) > 1$  when  $x > a$ ? (You may need to adjust your  $f(x)$  here, especially the sign. It's fine as long as you can still integrate it.)
- (e) We know  $\int_a^{+\infty} e^{-x^2} dx \leq \int_a^{+\infty} f(x)e^{-x^2} dx$ . Compute the new improper integral; is it finite?

(f) Now we just have to deal with  $\int_0^a e^{-x^2} dx$ . We can't do that integral exactly, but that's fine: you should be able to tell whether it's finite or not without doing any calculations. How?

(g) Does  $\int_0^{+\infty} e^{-x^2} dx$  converge?