

Math 1232: Single-Variable Calculus 2  
George Washington University Fall 2024  
Recitation 8

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**Problem 1.** Let  $(a_n) = (-6, 4, \frac{-8}{3}, \frac{16}{9}, \frac{-32}{27}, \dots)$ .

- (a) Find a closed-form formula for  $a_n$ .
- (b) Is there a real function  $f$  so that  $f(n) = a_n$ ?
- (c) What is  $\lim_{n \rightarrow \infty} a_n$ ? Why?

**Problem 2** (Factorials). (a) What is  $4!$ ? What is  $\frac{4!}{3!}$ ?

- (b) What is  $\frac{5!}{4!}$ ? What is  $\frac{5!}{3!}$ ?
- (c) Can you figure out what  $\frac{202!}{200!}$  is?

**Problem 3.** (a) Compute  $\lim_{n \rightarrow \infty} \frac{n}{n!}$ . Justify your answer.

- (b) Compute  $\lim_{n \rightarrow \infty} \frac{e^n}{n!}$ .
- (c) Now compute  $\lim_{n \rightarrow \infty} \frac{n^k}{n!}$ , where  $k > 0$  is a fixed integer.

**Problem 4.** Write out the first five terms of:

- (a)  $\sum_{k=1}^{\infty} \frac{(-2)^{k+1}}{3k}$
- (b)  $\sum_{k=1}^{\infty} \frac{k+1}{k!}$
- (c)  $\sum_{k=3}^{\infty} \frac{k+3}{k^2-k-2}$

**Problem 5.** Write in series/summation notation:

(a)  $1 + \frac{2}{3} + \frac{3}{5} + \frac{4}{7} + \dots$

(b)  $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} + \dots$

(c)  $2 + 7 + 14 + 23 + 34 + \dots$

**Problem 6.** (a) Use a telescoping series argument to write down a formula for  $\sum_{k=1}^n \frac{1}{k^2+3k+2}$ .

(b) Compute  $\sum_{k=1}^{\infty} \frac{1}{k^2+3k+2}$ .

(c) Use a telescoping series argument to write down a formula for  $\sum_{k=1}^n \frac{2}{k^2+2k}$ .

(d) Compute  $\sum_{k=1}^{\infty} \frac{2}{k^2+2k}$ .

(e) Use a telescoping series argument to write down a formula for  $\sum_{k=1}^n \ln\left(\frac{k+1}{k+3}\right)$ .

(f) Compute  $\sum_{k=1}^{\infty} \ln\left(\frac{k+1}{k+3}\right)$ .