Math 1232 Fall 2024
Single-Variable Calculus 2 Section 11
Mastery Quiz 9
Due Monday, October 28

This week's mastery quiz has two topics. Everyone should submit S7. If you already have a 2/2 on S6, you don't need to submit it again.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Monday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Secondary Topic 6: Differential Equations
- Secondary Topic 7: Sequences and Series

Name:

Recitation Section:

Name: Recitation Section:

S6: Differential Equations

(a) Find a general solution to the equation $y' = x^2 + 1 + x^2 y + y$.

Solution:

$$\begin{aligned} \frac{dy}{dx} &= (x^2+1)(1+y) \\ \frac{dy}{y} &= x^2 + 1\,dx \\ \ln|1+y| &= {}^{x^3}\!/_3 + x + C \\ 1+y &= e^{x^3/3+x+C} \\ y &= e^{x^3/3+x+C} - 1 = Ke^{x^3/3+x} - 1. \end{aligned}$$

(b) Find a (specific) solution to the initial value problem $y'/x - y = 1$ if $y(0) = 3$

Solution:

$$\begin{aligned} y'/x &= 1 + y \\ \frac{dy}{1+y} &= x\,dx \\ \ln|1+y| & x^2/2 + C \\ 1+y &= e^{x^2/2}e^C \\ y &= Ke^{x^2/2} - 1 \\ 3 &= K - 1 \Rightarrow K = 4 \\ y &= 4e^{x^2/2} - 1. \end{aligned}$$

S7: Sequences and Series

(a) Let $b_n = \frac{(n)!}{(n+2)!}$. Compute the first four terms of the sequence, and compute $\lim_{n\to\infty} b_n$.

Solution: $b_1 = {}^1\!/_6, b_2 = {}^2\!/_{24} = {}^1\!/_{12}, b_3 = {}^6\!/_{120} = {}^1\!/_{20}$, and $b_4 = {}^{24}\!/_{720} = {}^1\!/_{30}$.

We compute

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{(n+1)(n+2)} = \lim_{n\to\infty} \frac{1}{n^2+3n+2} = 0.$$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3 \cdot 5^{n-1}} =$

Solution: This is a geometric series with $a = \frac{2}{3}$ and $r = \frac{2}{5}$. So the sum is

$$\frac{2/3}{1-2/5} = \frac{2/3}{3/5} = \frac{10}{9}.$$

(c) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n+4}{n+3}\right) =$

Solution: This is the same as

$$\begin{aligned}\sum_{n=1}^{\infty} \ln(n+4) - \ln(n+3) &= \lim_{t\to\infty} \sum_{n=1}^{t} \ln(n+4) - \ln(n+3) \\ &= \lim_{t\to\infty} \big(\ln(5) - \ln(4)\big) + \big(\ln(6) - \ln(5)\big) + \cdots + \big(\ln(t+4) - \ln(t+3)\big) \\ &= \lim_{t\to\infty} \ln(t+4) - \ln(4) = \infty.\end{aligned}$$

Thus this sum diverges to infinity.