Math 1232: Single-Variable Calculus 2
George Washington University Fall 2024
Recitation 9

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Problem 1 (Geometric Series)**.** Compute:

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{2^k}{3^k}$

(b) $\displaystyle\sum_{k=2}^{\infty} \frac{(-5)^{k+2}}{2^{3k}}$

(c) $\frac{5}{2} + \frac{5}{4} + \frac{5}{8} + \frac{5}{16} + \dots$

(d) $\frac{-2}{3} + \frac{8}{9} + \frac{-32}{27} + \dots$

(e) $\frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81} + \dots$

Problem 2 (Infinite Decimals)**.** We want to find a rational representation of the infinite decimal $0.\overline{47}$. That is, we want to write $0.\overline{47} = \frac{p}{q}$ for integers p, q.

(a) First, what happens if we multiply $0.\overline{47}$ by 100?

(b) Using part (a), what can you tell about $(99) \cdot 0.\overline{47}$?

(c) Give a rational representation of $0.\overline{47}$.

(d) Now let's take a different approach. Write $0.\overline{47}$ as an infinite series.

(e) What kind of series is this? Can you use that fact to find a rational representation of $0.\overline{47}$?

(f) Now use the same logic to find a rational representation of $2.\overline{63}$.

Problem 3. For each of the following series, write a careful argument showing either that it converges or that it diverges. Think about exactly what test you want to use and why.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{5n^3-2}{3n^5-n}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{n^3\ln(n)+1}{n^4-7}$.

Problem 4 (Bonus)**.** Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(n^2+e^n)}{n^2}$ converge or diverge?