Math 1231 Midterm Solutions

Instructor: Jay Daigle

Problem 1 (M1)**.** Compute the following limits if they exist. Show enough work to justify your computation, or your claim that the limit does not exist.

(a)

$$\lim_{x\to 1} \frac{\sqrt{x}-3}{x-1} =$$

Solution:

$$\lim_{x\to 1} \frac{\sqrt{x}-3^{\nearrow -2}}{x-1_{\searrow 0}} = \pm\infty$$

(b)

$$\lim_{x\to 3} \frac{x^2-x-6}{x-3} =$$

Solution:

$$\begin{aligned} \lim_{x\to 3} \frac{x^2-x-6}{x-3} &= \lim_{x\to 3} \frac{(x-3)(x+2)}{x-3} \\ &= \lim_{x\to 3} x+2 = 5. \end{aligned}$$

(c)

$$\lim_{x\to +\infty} \frac{\sqrt{4x^2-3x+2}}{x+5} =$$

Solution:

$$\begin{aligned} \lim_{x\to +\infty} \frac{\sqrt{4x^2-3x+2}}{x+5} &= \lim_{x\to +\infty} \frac{\sqrt{4-3/x+2/x^2}}{1+5/x} \\ &= \frac{\sqrt{4-0+0}}{1+0} = 2. \end{aligned}$$

(d)

$$\lim_{x\to 0} \frac{\sin(2x)\tan(5x)}{3x^2} =$$

Solution:

$$\begin{aligned}
\lim_{x\to 0}\frac{\sin(2x)\tan(5x)}{3x^2} &= \lim_{x\to 0}\frac{\frac{\sin(2x)}{2x}\cdot 2x\frac{\sin(5x)}{\cos(5x)5x}5x}{3x^2}\\
&= \lim_{x\to 0}\frac{2x\cdot 5x}{\cos(5x)3x^2}\\
&= \lim_{x\to 0}\frac{10}{3\cos(5x)} = \frac{10}{3}.
\end{aligned}$$

Problem 2 (M2)**.** Compute the derivatives of the following functions using methods we have learned in class. Show enough work to justify your answers.

(a) Compute $\dfrac{d}{dx}\left(\dfrac{x^3+\sqrt[5]{x}-3}{\sec(x^2)+2}\right)^2$.

Solution:

$$\begin{aligned}
&\frac{d}{dx}\left(\frac{x^3+\sqrt[5]{x}-3}{\sec(x^2)+2}\right)^2\\
&= 2\frac{x^3+\sqrt[5]{x}-3}{\sec(x^2)+2}\cdot\frac{(3x^2+\frac{1}{5}x^{-4/5})(\sec(x^2)+2)-\sec(x^2)\tan(x^2)2x(x^3+\sqrt[5]{x}-3)}{(\sec(x^2)+2)^2}.
\end{aligned}$$

(b) Compute $\dfrac{d}{dx}\tan(x^2+\csc^2(x^3+\sin(x^5)))$.

Solution:

$$\begin{aligned}
&\frac{d}{dx}\tan(x^2+\csc(x^3+\sin(x^5)))\\
&= \sec^2(x^2+\csc(x^3+\sin(x^5)))\Big(2x-2\csc^2(x^3+\sin(x^5))\cot(x^3+\sin(x^5))\Big)\cdot(3x^2+\cos(x^5)\cdot 5x^4).
\end{aligned}$$

Problem 3 (S1)**.**

Suppose $f(x)=5x+4$, and we want an output of approximately 14. What input a should we aim for? Find a formula for δ in terms of ε so that if our input is $a\pm\delta$ then our output will be $14\pm\varepsilon$. Justify your answer.

Solution: We want an input of about $a=2$. Our output error will be $|5x+4-14|=|5x-10|=5|x-2|$. We want this to be less than ε, so we need

$$\begin{aligned}
5|x-2| &< \varepsilon\\
|x-2| &< \varepsilon/5
\end{aligned}$$

so we can take $\delta=\varepsilon/5$.

Problem 4 (S2)**. Directly from the definition of derivative,** compute the derivative of $f(x)=\sqrt{x-2}$ at $a=6$.

Solution:

$$\begin{aligned}
f'(6) &= \lim_{h\to 0}\frac{f(6+h)-f(6)}{h}\\
&= \lim_{h\to 0}\frac{\sqrt{4+h}-\sqrt{4}}{h}\\
&= \lim_{h\to 0}\frac{(4+h)-4}{h(\sqrt{4+h}+2)}\\
&= \lim_{h\to 0}\frac{1}{\sqrt{4+h}+2} = \frac{1}{2+2} = \frac{1}{4}.
\end{aligned}$$

Problem 5 (S3)**.** Give a formula for the linear approximation of the function $f(x) = \frac{x+2}{x-1}$ near the point $a = 2$. Use your formula to estimate $f(2.2)$.

Solution:

$$\begin{aligned}
f'(x) &= \frac{x-1-(x+2)}{(x-1)^2} \\
f'(2) &= \frac{-3}{1} = -3 \\
f(2) &= \frac{4}{1} = 4 \\
f(x) &\approx 4 - 3(x-2) \\
f(2.2) &\approx 4 - 3(2.2-2) = 4 - 3(.2) = 3.6.
\end{aligned}$$