Math 1231 Practice Final

Instructor: Jay Daigle

- You will have 120 minutes for this test.
- You are not allowed to consult books or notes during the test, but you may use a one-page, two-sided, handwritten cheat sheet you have made for yourself ahead of time. You must have written on the physical sheet you bring to the test in your own handwriting.
- You may not use a calculator. You may leave answers unsimplified, except you should compute trigonometric functions as far as possible.
- The exam has 14 problems, one on each mastery topic we've covered. The exam has 2 pages total.
- Each major topic has two parts, worth 10 points each. S9 is worth 10 points, and S10 has two parts, worth 10 points each. All of these questions are mandatory.
- You may submit *up to four* of the remaining secondary topics 1 through 8. Your final grade will be based on your *best two questions*, with a possible bonus point or two if you do well on the remaining questions. But we will not grade more than four, no matter how many you submit. Your final score will be out of 130 points.
- If you perform well on a question on this test it will update your mastery scores. Achieving a 18/20 on a major topic or on S10, or 9/10 on another secondary topic, will count as getting a 2 on a mastery quiz.
- Read the questions carefully and make sure to answer the actual question asked. Make sure to justify your answers—math is largely about clear communication and argument, so an unjustified answer is much like no answer at all.

  When in doubt, show more work and write complete sentences.
- If you need more paper to show work, I have extra at the front of the room.
- Good luck!

Problem 1 (M1). (a) Compute $\lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25}$

(b) Compute $\lim_{x \to -\infty} \frac{3x}{\sqrt{4x^2 + 3}}$

Problem 2 (M2). (a) Find $\frac{d}{dx} \sqrt[4]{\frac{x^3 + \cos(x^2)}{\sin(x^3) + 1}}$

(b) Find $h(x) = \sec^3\left(\frac{x^3 + 1}{x - 1}\right)$.

Problem 3 (M3). (a) The function $f(x) = 3x^4 - 20x^3 + 24x^2 + 7$ has absolute extrema either on the interval $[0, 5]$ or on the interval $(0, 5)$. Pick one of those intervals, explain why f has extrema on that interval, and find the absolute extrema.

(b) Find and classify the critical points of $g(x) = \frac{2x - 1}{x^2 + 2}$.

Problem 4 (M4). (a) Compute $\int \sin^4(t)\cos(t)\,dt$

(b) **By explicitly changing the bounds of the integral**, compute $\int_0^4 x^3\sqrt{9+x^2}\,dx$.

Problem 5 (S9). Using **only the definition of Riemann sum** and your knowledge of limits, compute the exact area under the curve x^2+x^3 between $x=1$ and $x=3$.

Problem 6 (S10). (a) A spring with natural length of 8 inches takes 6 pounds of force to stretch to 10 inches. Compute the work done by stretching the spring from 12 inches to 16 inches. What units will this integral output?

(b) Sketch the region bounded by the curves $y=\sqrt{x}, x=4, y=0$, and find the volume when this region is revolved around the line $x=-1$.

Do not submit more than four of the remaining questions. Your grade will be based on your best two. These questions can also help raise your mastery scores.

Problem 7 (S1). Suppose $f(x)=x^2+3$, and we want an output of approximately 19. If we want our input to be positive, what input a should we aim for? Find a δ so that if our input is $a\pm\delta$ then our output will be 19 ± 1. Explain how you found this δ and why it should give us what we want.

Problem 8 (S2). **Directly from the definition**, compute $f'(1)$ where $f(x)=\sqrt{x+3}$.

Problem 9 (S3). If $f(x)=\sqrt{x}+\tan(\pi x)$, use a linear approximation centered at 4 to estimate $f(4.1)$.

Problem 10 (S4). Suppose that if a car travels at v miles per hour then its fuel efficiency is $F(v)=8+1.3v-.015v^2$ miles per gallon.

(i) What does the derivative $F'(v)$ represent, and what are its units?

(ii) Compute $F'(60)$. What does this tell you?

Problem 11 (S5). Find a formula for y' in terms of x and y if $x^8+x^4+y^4+y^6=1$.

Problem 12 (S6). A cone with height h and base radius r has volume $\frac{1}{3}\pi r^2 h$. Suppose we have an inverted conical water tank, of height 4m and radius 6m. Water is leaking out of a small hole at the bottom of the tank. If the current water level is 2m and the water level is dropping at $\frac{1}{9\pi}$ meters per minute, what volume of water leaks out every minute?

Problem 13 (S7). Let $j(x)=x^4-14x^2+24x+6$. We can compute $j'(x)=4(x+3)(x-1)(x-2)$ and $j''(x)=4(3x^2-7)$. Sketch a graph of j.

Your answer should discuss the domain, asymptotes, limits at infinity, critical points and values, intervals of increase and decrease, and concavity.

Problem 14 (S8). Suppose you are running a toy shop. It costs $C(x)=200+10x$ dollars to produce x toys in a day, and you make a revenue of $R(x)=26x-.2x^2$ dollars if you sell x toys in a day. How many toys should you produce per day to maximize your profit? Make sure you give a complete sentence for your answer.