Math 1231 Practice Midterm

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- This is a practice test. It is deliberately somewhat harder than I expect the real test to be, because that's good practice. These are the same instructions you will see on the real test.
- You will have 75 minutes for this test.
- You are not allowed to consult books or notes during the test, but you may use a one-page, one-sided, handwritten cheat sheet you have made for yourself ahead of time.
- You may not use a calculator. You may leave answers unsimplified, except you should compute trigonometric functions as far as possible.
- The exam has 6 problems, one on each mastery topic we've covered. The exam has 5 pages total.
- The question on S7 is worth 15 points. The other questions are worth 10 points each. The whole test is scored out of 75 points.
- Read the questions carefully and make sure to answer the actual question asked. Make sure to justify your answers—math is largely about clear communication and argument, so an unjustified answer is much like no answer at all.

  When in doubt, show more work and write complete sentences.
- If you need more paper to show work, I have extra at the front of the room.
- Good luck!

Problem 1 (M3).

1. Find and classify all the critical points of $f(x) = (x-5)\sqrt[3]{x^2}$. [Note: this is quite hard but it's good practice.]

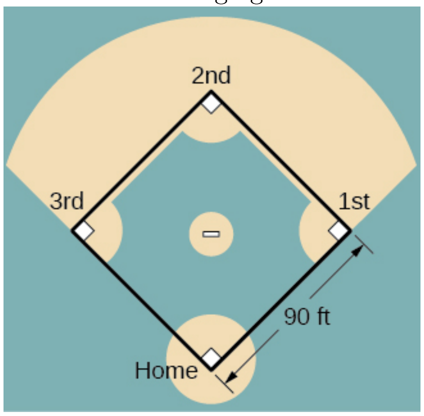
2. The function $g(x) = (x^2 - 3x)\sqrt[3]{x-3}$ has absolute extrema either on $(-4, -1)$ or on $[1, 4]$. Pick one of those intervals, explain why g has extrema on that interval, and find the absolute extrema.

Problem 2 (S4)**.** Suppose that $Q(p) = 3p^2 + 10p - 100$ is the number of widgets you can buy at a price of p dollars.

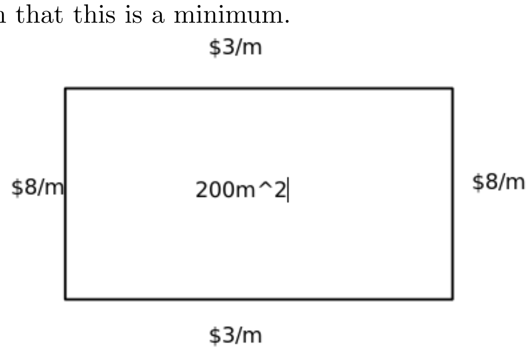
1. What are the units of $Q'(p)$? What does it represent physically? What does it mean if $Q'(p)$ is big?
2. Calculate $Q'(10)$. What does this tell you physically? What physical observation could you make to check your calculation?

Problem 3 (S5)**.** Find a tangent line to the curve given by $x^4 - 2x^2y^2 + y^4 = 16$ at the point $(\sqrt{5}, 1)$.

Problem 4 (S6)**.** Consider this baseball diamond, which is a square with 90ft sides. A batter hits the ball and runs from home toward first base at a speed of 22ft/s. At what rate is the distance between the runner and second base changing when the runner has run 30ft?



Problem 5 (S8)**.** We want to build a rectangular fence that will enclose 200m^2. One pair of parallel sides cost \$3/m and the other pair costs \$8/m. What dimensions minimize the cost of the fence? Justify your claim that this is a minimum.



Problem 6 (S7)**.** Let $f(x) = \dfrac{x^3 - 2}{x^4}$. We compute that $f'(x) = \dfrac{8 - x^3}{x^5}$ and $f''(x) = \dfrac{2x^3 - 40}{x^6}$. Sketch a graph of f.

Your answer should discuss the domain, asymptotes, roots, limits at infinity, critical points and values, intervals of increase and decrease, and concavity.