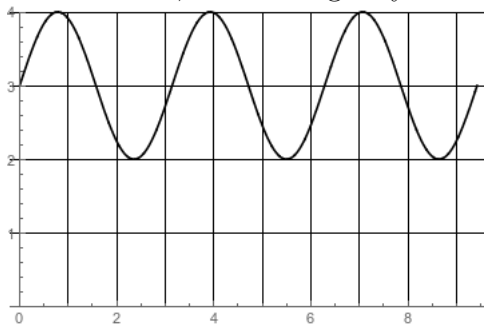


Math 1231-13: Single-Variable Calculus 1
George Washington University Spring 2024
Recitation 11

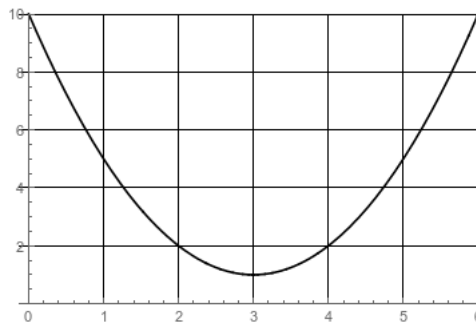
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Problem 1. For the following curves, find an upper bound and a lower bound for the area under the curve, and then give your best estimate for the actual area.



(between 0 and 9; ignore the trailing bit off the right edge)



(between 0 and 6; ignore the trailing bit off the right edge)

Problem 2. Consider the function $f(x) = \sqrt{1 - x^2}$ between $x = 0$ and $x = 1$.

- Estimate the area using two rectangles with right endpoints. Is this an upper bound, a lower bound, or neither?
- Estimate the area using two rectangles with left endpoints. Is this an upper bound, lower bound, or neither?
- Find an upper bound using four rectangles.
- Find a lower bound using four rectangles.
- Can you guess what the area under the curve is exactly? (Hint: what does the graph look like?)

Problem 3. Consider the function $g(x) = x^3$ between $x = 0$ and $x = 1$.

- (a) Estimate the area using two rectangles with right endpoints. Is this an upper bound, a lower bound, or neither?
- (b) Estimate the area using two rectangles with left endpoints. Is this an upper bound, lower bound, or neither?
- (c) Find an upper bound using four rectangles.
- (d) Find a lower bound using four rectangles.
- (e) Find a formula using right endpoints to estimate the area using n rectangles, in summation form.
- (f) Use your summation rules to get a closed-form formula, with no summation signs in it.
- (g) Take a limit to find the exact area under this curve. (Use your summation rules!)

Problem 4. Suppose we know that $\int_2^4 f(x) dx = 3$, $\int_4^6 f(x) dx = 5$, and $\int_2^6 g(x) dx = -2$. Compute the following integrals, justifying your answers:

- (a) $\int_2^4 3f(x) dx$?
- (b) $\int_2^6 f(x) - g(x) dx$?
- (c) $\int_6^4 f(x) - 3 dx$?

Problem 5. (a) Use the Fundamental Theorem of Calculus to compute $\frac{d}{dx} \int_2^x \sqrt{t^5 - t} dt$.

- (b) Compute $\frac{d}{dx} \int_x^5 s^5 + \cos(s^2) ds$. What rule did you have to use here other than the FTC?
- (c) Compute $\frac{d}{dx} \int_{-3}^{x^2} \sqrt{t^3 + 1} dt$. What rule did you have to use here other than the FTC?

Problem 6. We want to find $\frac{d}{dx} \int_{3x}^{x^3} \sqrt[3]{x+1} dx$. Unfortunately we can't apply the Fundamental Theorem of Calculus directly.

- (a) This integral has variables in both the upper and lower bounds. Can you split it into multiple integrals, each of which has only one variable in a bound?
- (b) To use the FTC we need the variable as the *upper* bound of each integral. How can we do that?

- (c) Now for each integral you have set up, carefully take the derivative, paying attention to the chain rule.
- (d) Combine this work to answer the original question.