

Math 1231-13: Single-Variable Calculus 1
George Washington University Spring 2024
Recitation 13

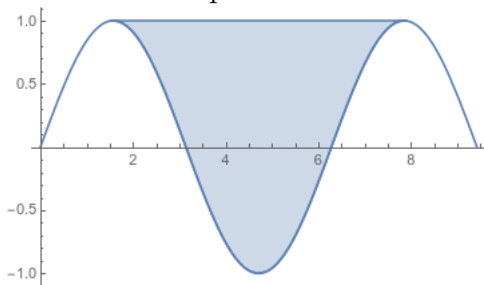
Jay Daigle

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Problem 1. We want to find the area of the region bounded by $y = x^2 + 1$, $y = 17 - x^2$, and $x = 1$, taking the side with $x \geq 1$.

- (a) Sketch the region in question. Based on the picture, would you rather integrate with respect to x or to y ? Discuss this with someone near you.
- (b) Set up an integral to compute this region, integrating with respect to x .
- (c) Set up an integral to compute this region, integrating with respect to y .
- (d) Which of these integrals do you prefer? Pick one and compute it.

Problem 2. Compute the total area of the “valley” between two peaks of the sine function.



Problem 3. For each of the following functions, figure out the units of $\int f(x) dx$. What is this integral computing:

- (a) $f(x)$ gives acceleration in meters per second squared as a function of time in seconds.

- (b) $f(x)$ gives tension in pounds per inch, as a function of how many inches along a material you are.
- (c) $f(x)$ gives the pressure exerted by a gas (in newtons per square meter), as a function of the volume in cubic meters. (Imagine a piston moving out to expand a chamber full of gas under pressure.)
- (d) $f(x)$ gives density in kilograms per meter, as a function of how many meters along a steel rod you are.
- (e) $f(x)$ gives resistance in volts per ampere as a function of how many amperes you run through a wire.

Problem 4. Find the average value of the function $\frac{x}{(x^2 + 1)^2}$ for $1 \leq x \leq 3$.

Problem 5. Suppose the demand for pizzas is $D(q) = 25 - .0001q^2$ and the supply is $S(q) = 10 + .02q$.

- (a) How many pizzas will be sold, and at what price?
- (b) What is the consumer surplus?
- (c) What is the producer surplus?
- (d) What is the total surplus?

Problem 6. A 12in spring is stretched to 15in by a force of 75lbs.

- (a) What is the spring constant? What units does it have?
- (b) What is the function that gives force as a function of position? what units does it have?
- (c) What is the work done by stretching the spring from 16in to 20in? What units are your answer in?