

Math 1231-13: Single-Variable Calculus 1  
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Recitation 2

Jay Daigle

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**Problem 1.** Let  $f(x) = 5x + 2$ . We want to use an  $\varepsilon - \delta$  argument to compute  $\lim_{x \rightarrow 2} f(x)$ .

- (a) If  $x$  is about 2, what should  $f(x)$  be?
- (b) Write down expressions using absolute value for the input and output errors.
- (c) If we want  $\varepsilon = 1$ , what does  $\delta$  need to be?
- (d) Find a formula for  $\delta$  in terms of  $\varepsilon$  (same form as  $\delta = \varepsilon/3$  or  $\delta = \varepsilon$ ).
- (e) Try to write a full proof.

But we mostly want to practice the way we actually compute limits.

**Problem 2** (Warmup). Let  $f(x) = \frac{x^2 + \sin(x) + 3}{x^2 - x - 2}$ .

- (a) Where is  $f$  continuous? Where is it discontinuous?
- (b) What is  $\lim_{x \rightarrow 0} f(x)$ ?

**Problem 3.** Let  $f(x) = \frac{x-1}{x^2-1}$ .

- (a) What is  $f(2)$ ? Is  $f$  continuous at 2?
- (b) What is  $\lim_{x \rightarrow 2} f(x)$ ?
- (c) What is  $f(1)$ ? Is  $f$  continuous at 1?

(d) What function can we find that's almost the same as  $f$ , but defined and continuous at 1? (Is this function the same as  $f$ ?)

(e) What is  $\lim_{x \rightarrow 1} f(x)$ ?

**Problem 4.** Let  $g(x) = \frac{(x+1)^2-1}{x+2}$ .

(a) Is  $g$  continuous where it's defined? Where is it undefined?

(b) Can you find a function that's almost identical to  $g$  but continuous everywhere?

(c) What is  $\lim_{x \rightarrow -2} g(x)$ ?

**Problem 5.** Let  $h(x) = \frac{x-1}{\sqrt{5-x}-2}$ .

(a) Is this function continuous where it's defined? Where is it undefined?

(b) We can factor an  $x - 1$  out of the top, but we can't obviously factor one out of the bottom. We need to use an algebraic trick make the  $x - 1$  appear. What tricks do we have that might work?

(c) What is  $\lim_{x \rightarrow 1} h(x)$ ?