# Math 1231 Spring 2024
# Single-Variable Calculus I Section 11
# Mastery Quiz 3
# Due Tuesday, February 6

This week's mastery quiz has only one topic. Everyone should submit topic M1 (even if you got a 2 last week; your best two scores count). Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material.

Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Thursday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

**Topics on This Quiz**

- Major Topic 1: Computing Limits

## Name:

## Recitation Section:

# Major Topic 1: Computing Limits

(a) $\lim_{x\to 0} \dfrac{\sin(5x^2) + \tan^2(3x)}{x^2} =$

**Solution:**

$$\begin{aligned}
\lim_{x\to 0} \frac{\sin(5x^2) + \tan^2(3x)}{x^2} &= \lim_{x\to 0} \frac{\frac{\sin(5x^2)}{5x^2} 5x^2 + \frac{\sin(3x)}{3x} \cdot (3x) \cdot \frac{\sin(3x)}{3x} \cdot (3x) \cdot \frac{1}{\cos^2(3x)}}{x^2} \\
&= \lim_{x\to 0} \frac{5x^2 + \frac{9x^2}{\cos^2(3x)}}{x^2} \\
&= \lim_{x\to 0} 5 + \frac{9}{\cos^2(3x)} = 5 + 9 = 14.
\end{aligned}$$

by the Small Angle Approximation.

(b) $\lim_{x\to +\infty} \dfrac{3x^2 + 2x + 1}{\sqrt{x^4 - x^2 + x}} =$

**Solution:**

$$\begin{aligned}
\lim_{x\to +\infty} \frac{3x^2 + 2x + 1}{\sqrt{x^4 - x^2 + x}} &= \lim_{x\to +\infty} \frac{3 + 2/x + 1/x^2}{1/\sqrt{x^4}\sqrt{x^4 - x^2 + x}} \\
&= \lim_{x\to +\infty} \frac{3 + 2/x + 1/x^2}{\sqrt{1 - 1/x^2 + 1/x^3}} \\
&= \frac{3 + 0 + 0}{\sqrt{1 - 0 + 0}} = 3.
\end{aligned}$$

(c) $\lim_{x\to 4^+} \dfrac{x+1}{x-4} =$

**Solution:** The limit of the top is 5 and the limit of the bottom is 0, so the limit is $\pm\infty$. Since the bottom will always be positive as we approach from the right, the overall limit is in fact $+\infty$.