

Math 1231-13: Single-Variable Calculus 1
George Washington University Spring 2024
Recitation 4

Jay Daigle

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Problem 1. Let $f(x) = x^3$. We want to find a formula for the derivative of this function at any given point.

- (a) Write down a formula for $f'(a)$ using the $h \rightarrow 0$ limit formulation. What does the numerator mean? What does the denominator mean?
- (b) Use your formula from part (a) to compute the derivative.
- (c) Now write down a formula for $f'(a)$ using the $x \rightarrow a$ limit formulation. Does this look easier or harder than the formula from part (a), and why? What does the numerator mean? What does the denominator mean?
- (d) Use the formula from part (c) to compute the derivative. You should get the same answer you got in part (b).
- (e) Which method was faster? Which method was easier?

Problem 2. Let $g(x) = \sqrt[3]{x}$.

- (a) Write down a limit formula to compute the derivative of g at 0.
- (b) What is $g'(0)$? What does this tell you?
- (c) Now write down a limit formula to compute the derivative of $p(x) = \sqrt[3]{x^2}$.
- (d) What is this limit? What does that tell you?

- (e) Write down a limit formula to compute the derivative of g at a when $a \neq 0$.
- (f) (Bonus) Can you compute this limit? What do you have to do here? (It's not obvious, but there's an algebraic trick from Day 1 that can help us.)

Problem 3. Let $f(x) = \sqrt{x^2 - 4}$.

- (a) Set up a limit expression to calculate $f'(x)$. Do you think $h \rightarrow 0$ or $x \rightarrow a$ will be easier here?
- (b) Compute $f'(x)$.
- (c) Where is f differentiable? Where is it not differentiable?

Problem 4. Let $g(x) = \frac{1}{x+3}$.

- (a) Write down a limit expression to compute $g'(2)$. Be careful with order of operations and parentheses!
- (b) Now compute $g'(2)$.
- (c) Write a limit expression to compute $g'(x)$. Again, make sure you get your order of operations right.
- (d) Compute $g'(x)$.