

Math 1231-13: Single-Variable Calculus 1
George Washington University Spring 2024
Recitation 4

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Problem 1. (a) Let $h(x) = \tan^2(x)$. Find functions f and g so that $h(x) = (f \circ g)(x)$.

(b) Compute $f'(x)$ and $g'(x)$. Use that info to compute $h'(x)$.

(c) Now let $h(x) = \tan(x^2)$. Find functions f and g so that $h(x) = (f \circ g)(x)$.

(d) Compute $f'(x)$ and $g'(x)$. Use that information to compute $h'(x)$.

Problem 2. Consider the function $\sec^2(x^2 + 1)$

(a) Find functions f and g so that $(f \circ g)(x) = \sec^2(x^2 + 1)$.

(b) Talk to the people next to you. Did they pick the same f and g that you did? Can you find a different pair of functions f and g that also work?

(c) Find functions f, g, h so that $(f \circ g \circ h)(x) = \sec^2(x^2 + 1)$.

(d) Compute $f', g',$ and h' .

(e) What is $\frac{d}{dx} \sec^2(x^2 + 1)$?

Problem 3. Find

$$\frac{d}{dx} \frac{\sin(x^2) + \sin^2(x)}{x^2 + 1}$$

Problem 4. (a) Compute

$$\frac{d}{dx} \sqrt{\frac{\sqrt{x} + 1}{(\cos x + 1)^2}}$$

(b) Find

$$\frac{d}{dx} \tan^4(\sqrt[3]{x^5 + x^3 + 2} + 1).$$

Problem 5 (Bonus). Calculate

$$\frac{d}{dx} \left(\frac{\sin^2\left(\frac{x^2+1}{\sqrt{x-1}}\right) + \sqrt{x^3-2}}{\cos(\sqrt{x^2+1}+1) - \tan(x^4+3)} \right)^{5/3}$$

Problem 6 (Geometric Series). Another function it's sometimes important to approximate is the "geometric series" formula $f(x) = \frac{1}{1-x}$, near $x = 0$.

- (a) What is $f'(x)$?
- (b) Find a linear approximation for $f(x)$ near $x = 0$.
- (c) Use this formula to estimate $\frac{1}{.9}$ and $\frac{1}{1.01}$. Do these answers make sense?
- (d) Use your formula to estimate $\frac{1}{1.5}$ and $\frac{1}{10.5}$. Do these answers make sense?
- (e) Use your formula to estimate $f(-1)$ and $f(1)$. Do these answers make sense?