Math 1231 Spring 2024
Single-Variable Calculus I Section 11
Mastery Quiz 6
Due Tuesday, February 27

This week's mastery quiz has three topics. Everyone should submit topics S4. If you already have a 4/4 on M2 or a 2/2 on S3 (check Blackboard!) you don't need to submit them again.

Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Thursday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 2: Computing Derivatives
- Secondary Topic 3: Linear Approximation
- Secondary Topic 4: Rates of Change

Name:

Recitation Section:

Major Topic 2: Computing Derivatives

(a) Compute $\frac{d}{dx}\frac{\sin(\csc(x^2+1))}{x^4+\cos(x)} =$

Solution:

$$\frac{\big(\cos(\csc(x^2+1))(-\csc(x^2+1)\cot(x^2+1))2x\big)(x^4+\cos(x)) - (4x^3-\sin(x))\sin(\csc(x^2+1))}{(x^4+\cos(x))^2}.$$

(b) Compute $\frac{d}{dx}\cos^2\left(\tan^2\left(\sec^2(\sqrt{x}+x)\right)\right)$.

Solution:

$$\begin{aligned} 2\cos\left(\tan^2\left(\sec^2(\sqrt{x}+x)\right)\right)\left(-\sin\left(\tan^2\left(\sec^2(\sqrt{x}+x)\right)\right)\right) \\ \cdot 2\tan\left(\sec^2(\sqrt{x}+x)\right)\sec^2\left(\sec^2(\sqrt{x}+x)\right) \\ \cdot 2\sec(\sqrt{x}+x)\sec(\sqrt{x}+x)\tan(\sqrt{x}+x)\left(\frac{1}{2x}+1\right) \end{aligned}$$

Secondary Topic 3: Linear Approximation

(a) Use linear approximation to estimate $\sqrt[4]{14}$.

Solution: We take $f(x) = \sqrt[4]{x}$, and take $a = 16$. Then

$$\begin{aligned} f'(x) &= \frac{1}{4}x^{-3/4} \\ f'(16) &= \frac{1}{4}(16)^{-3/4} = \frac{1}{4\cdot 8} = \frac{1}{32} \\ f(x) &\approx f(a) + f'(a)(x-a) = 2 + \frac{1}{32}(14-16) = 2 - \frac{1}{16} = \frac{31}{16}. \end{aligned}$$

(b) Write the equation for the tangent line to $g(x) = 2x - \tan(x)$ at the point $a = \pi$.

Solution:

$$\begin{aligned} g(\pi) &= 2\pi - 0 = \pi \\ g'(x) &= 2 - \sec^2(x) \\ g'(\pi) &= 2 - 1 = 1 \\ y &= 2\pi + (x - \pi) \end{aligned}$$

Secondary Topic 4: Rates of Change

(a) Let $F(x) = \frac{1}{x} + 1$ be the amount of pressure exerted on a beam in pounds per square inch at a point x inches to the right of its left end.

(i) What are the units of $F'(x)$? What does $F'(x)$ represent physically? What would it mean if $F'(x)$ is big?

(ii) Compute $F'(5)$. What does this tell you physically? What physical observation could you make to check your calculation?

Solution:

(i) The derivative $F'(x)$ has units pounds per square inch per inch, or pounds per cubic inch. $F'(x)$ is the rate at which pressure is increasing as you move to the right along the stick. If $F'(x)$ is big, that means that moving along the stick a little bit will increase the pressure by a lot.

(ii) $F'(x) = -1/x^2$ so $F'(5) = -1/25$. This means that if we are five inches to the right of the endpoint, moving one more inch to the right should decrease the pressure by about $1/25$ of a pound per square inch.

(b) Suppose the vertical position of a weight on a spring in inches is given as a function of time in seconds by the formula $h(t) = \cos(2t)$.

(i) When is the velocity zero?

(ii) When is the acceleration zero?

Solution:

(i) $p'(t) = -2\sin(2t)$ so the velocity is zero when $\sin(2t) = 0$. This happens when $2t = 0, \pi, 2\pi, \ldots$, and thus when $t = 0, \pi/2, \pi, 3\pi/2, \ldots$. In other words, at $n\pi/2$.

(ii) $p''(t) = -4\cos(2t)$ is zero when $2t = \pi/2, 3\pi/2, \ldots$, and thus when $t = \pi/4, 3\pi/4, 5\pi/4, \ldots$. We could say $t = (2n+1)\pi/4$.