

Math 1231-13: Single-Variable Calculus 1
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Recitation 6

Jay Daigle

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Problem 1 (Geometric Series). Another function it's sometimes important to approximate is the "geometric series" formula $f(x) = \frac{1}{1-x}$, near $x = 0$.

- (a) What is $f'(x)$?
- (b) Find a linear approximation for $f(x)$ near $x = 0$.
- (c) Use this formula to estimate $\frac{1}{.9}$ and $\frac{1}{1.01}$. Do these answers make sense?
- (d) Use your formula to estimate $\frac{1}{1.5}$ and $\frac{1}{1.05}$. Do these answers make sense?
- (e) Use your formula to estimate $f(-1)$ and $f(1)$. Do these answers make sense?

Problem 2. (a) Use the binomial approximation to estimate $\sqrt{2}$ and $\sqrt[3]{2}$.

- (b) Use the binomial approximation to estimate $\sqrt{17}$. (Remember: 17 is not close to 1! You need to be slightly clever here.)
- (c) Can you find a formula to approximate $(1+x^n)^\alpha$ for a real number α ?
- (d) What does this tell us about $\sqrt{1+x^2}$?

Problem 3 (Bonus). Find a formula to approximate $f(x) = x^3 + 3x^2 + 5x + 1$ near $a = 0$. What do you notice? Why does that happen?

Problem 4. Suppose that $p(t) = 10 - 2t$ is momentum (in kg m/s) of a ball thrown directly upwards, as a function of time (in seconds).

- (a) What units does the derivative $p'(t)$ take as input? What units are its output? (Do you know of any physical quantity that's represented by those units?)
- (b) What does the derivative $p'(t)$ represent physically? What would it mean for $p'(t)$ to be big, or small?
- (c) Calculate $p'(3)$. What does this tell you? What physical observation could you measure to check if your calculation was correct?

Problem 5. Suppose the cost of buying m machines is $C(m) = 500 + 10m + .05m^2$. There's some start-up cost to having any machines at all; then each machine costs a bit more than the previous one.

- (a) What are the units of the inputs to the function C ? What are the units of the outputs?
- (b) What is $C(1)$? $C(10)$? $C(100)$?
- (c) Find a formula for $C'(m)$. What are the units of the input and output to $C'(m)$?
- (d) What is $C'(10)$? How should we interpret this number?
- (e) What is the *average* cost per machine when you have ten machines? How does this compare to your previous answer?
- (f) What is $C''(m)$? What are the units? What is $C''(10)$ and how should we interpret it?

Problem 6 (Bonus). Let $Q(p) = 10000 - 10p$ give the number of widgets you can sell at a given price p .

- (a) If you set a price of \$100, how many widgets will you be able to sell? What if you set a price of \$1000?
- (b) What is the derivative of Q ? What are its units?
- (c) What is $Q'(100)$ and what does that tell you?

Problem 7. Find the tangent line to $y = 6 \cos x$ at $(\pi/3, 3)$.

Problem 8. Find an equation for a tangent line to $y \cos(x) = 1 + \sin(xy)$ at the point $(0, 1)$.

Problem 9 (Bonus). (a) If $\sqrt{xy} = x^2y - 2$, find a formula for $\frac{dy}{dx}$ in terms of x and y .

- (b) Find an equation of the tangent line at the point $(1, 4)$.