

Math 1231 Spring 2024  
Single-Variable Calculus I Section 11  
Mastery Quiz 9  
Due Tuesday, March 26

This week's mastery quiz has three topics. Everyone should submit all three (sorry).

Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Thursday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

**Topics on This Quiz**

- Major Topic 3: Optimization and Extrema
- Secondary Topic 7: Curve Sketching
- Secondary Topic 8: Physical Optimization

**Name:**

**Recitation Section:**

### Major Topic 3: Optimization and Extrema

- (a) The function  $f(x) = \frac{x^2 + 5}{x + 2}$  has absolute extrema either on the interval  $[-3, 0]$  or on the interval  $[0, 3]$ . Pick one of those intervals, explain why  $f$  has extrema on that interval, and find the absolute extrema.
- (b) Find and classify the critical points of  $f(x) = \sqrt[3]{x^3 - 3x}$ .

### Secondary Topic 7: Curve Sketching

Let  $g(x) = \frac{x^2 - 7}{x^2 - 4}$ .

We can compute that  $g'(x) = \frac{6x}{(x+2)^2(x-2)^2}$  and  $g''(x) = \frac{-6(3x^2 + 4)}{(x^2 - 4)^3}$ .

Sketch a graph of the function  $g(x)$ . Your answer should discuss the domain, asymptotes, roots, limits at infinity, critical points and values, intervals of increase and decrease, points of inflection, and concavity.

### Secondary Topic 8: Physical Optimization

Suppose that a company that produces and sells  $x$  units of a product makes a revenue of  $R(x) = 260x - 9x^2/10$  and has costs given by  $C(x) = 1000 + 100x + x^2/10$ . What is the maximum profit that can be made (where profit is revenues minus costs)?