

Math 1231-13: Single-Variable Calculus 1  
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Recitation 9

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**Problem 1.** Let  $g(x) = x \tan(x)$ . We want to sketch a graph of  $g$ .

- (a) What is the domain of  $g$ ?
- (b) For simplicity, let's just look at  $[-\pi/2, \pi/2]$ . What can you say about any asymptotes it has?
- (c) Does this function have any roots you can find?
- (d)  $g'(x) = \frac{\sin(x) \cos(x) + x}{\cos^2(x)}$ . What are the critical points?  
(Hint: when is  $\sin(x) \cos(x)$  positive and when is it negative?)
- (e) What are the critical values?
- (f) Where is  $g$  increasing and decreasing? Does it have maxima or minima?
- (g)  $g''(x) = 2 \sec^2(x)(1 + x \tan(x))$ . Where are the potential points of inflection, and what are their values? Where is  $h$  concave up and down?
- (h) Sketch the graph.

**Problem 2.** Let  $h(x) = \frac{x+2}{x-1}$ . We want to sketch a graph of  $h$ .

- (a) What is the domain of  $h$ ? What can you say about any asymptotes it has?
- (b) Does this function have any roots? Where?

- (c) What happens as  $x$  approaches  $+\infty$ ?  $-\infty$ ?
- (d)  $h'(x) = -3(x - 1)^{-2}$ . What are the critical points and values?
- (e) Where is  $h$  increasing and decreasing? Does it have maxima or minima?
- (f)  $h''(x) = 6(x - 1)^{-3}$ . Where are the potential points of inflection? Where is  $h$  concave up and down?
- (g) Sketch the graph.

**Problem 3** (Bonus). Let  $g(x) = x^5 - 4x^3 + 4x + 7$ . We want to sketch a graph of  $g$ .

- (a) What is the domain of  $g$ ? What can you say about any asymptotes it has?
- (b) Does this function have any roots you can find?
- (c) What happens as  $x$  approaches  $+\infty$ ?  $-\infty$ ?
- (d)  $g'(x) = 5x^4 - 12x^2 + 4$ . What are the critical points?  
(Hint: if we set  $u = x^2$  this becomes a quadratic, and we can factor it.)
- (e) What are the critical values?  
(Hint:  $g(x) = 7 + x(x^4 - 4x^2 + 4) = 7 + x(u^2 - 4u + 4)$ .)
- (f) Where is  $g$  increasing and decreasing? Does it have maxima or minima?
- (g)  $g''(x) = 20x^3 - 24x$ . Where are the potential points of inflection, and what are their values? Where is  $h$  concave up and down?
- (h) Sketch the graph.