# Math 1232 Spring 2024
# Single-Variable Calculus 2 Section 12
# Mastery Quiz 1
# Due Tuesday, January 23

This week's mastery quiz has one topic. Please submit your best attempt at answering the questions on that topic, and try to demonstrate your mastery of the underlying material. (And don't worry if your answers aren't completely solid; this is a quiz but it's also a learning experience, and you'll get another try next week.)

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Tuesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

**Topics on This Quiz**

- Secondary Topic 1: Invertible Functions

## Name:

## Recitation Section:

# S1: Invertible Functions

(a) Is $f(x) = x^2 + x$ invertible or not? Justify your answer.

**Solution:** We have $f(-1) = f(0) = 0$ so this function is not one-to-one, and thus not invertible.

(b) Give an exact solution (with no decimals) for the equation $\log_3(2x + 5) = 3$. Do not use a calculator.

**Solution:**

$$\begin{aligned} \log_3(2x+5) &= 3 \\ 2x+5 &= 3^3 \\ 2x &= 27-5 \\ x &= \frac{22}{2} = 11. \end{aligned}$$

(c) Let $h(x) = \sqrt{x^3 + x + 6}$. Compute $(h^{-1})'(6)$.

**Solution:** By the Inverse Function Theorem, we know that

$$(h^{-1})'(6) = \frac{1}{h'(h^{-1}(6))}.$$

Guess and check shows that $h(3) = 6$ so $h^{-1}(6) = 3$. And we know that

$$h'(x) = \frac{1}{2}(x^3 + x + 6)^{-1/2}(3x^2 + 1)$$

and thus

$$h'(3) = \frac{1}{2}(36)^{-1/2}(28) = \frac{14}{6}.$$

Thus

$$(h^{-1})'(4) = \frac{1}{14/6} = \frac{3}{7}.$$