Math 1232: Single-Variable Calculus 2
George Washington University Spring 2024
Recitation 10

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Problem 1. For each of the following series, write a careful argument showing either that it converges or that it diverges. Think about exactly what test you want to use and why.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{5n^3-2}{3n^5-n}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{n^3\ln(n)+1}{n^4-7}$.

Problem 2. Analyze the convergence of the following series. Write clean arguments that establish whether they diverge, converge conditionally, or converge absolutely. Think about what tools/tests you want to use, and why.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n!}{2^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n^2+e^n)}{n^2}$

(c) $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^{n-1}}{\ln(n)}$

Problem 3. Let r be a real number. Does $\sum_{n=1}^{\infty} \frac{r^n}{n!}$ converge? What test do we want to use, and why? Does the answer depend on the value of r?

Problem 4. In class we showed that $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ converges to a number between 1 and 1/2. I claimed that it converged to $\ln 2$. But this convergence is conditional, and that matters.

(a) Write out the first twelve terms of this series.

(b) Reorganize them so that you have the same collection of numbers add one and then subtract two, then add one, then subtract two, and so on. (You'll have some extras left over and that's fine; remember you have an infinite list of terms.)

(c) What does each triplet look like? Can you simplify that somehow so it looks like something we recognize? (Hint: what happens if you combine the first two terms of a triplet?)

(d) Can you figure out what this sequence of partial sums converges to?