Math 1232 Spring 2024
Single-Variable Calculus 2 Section 12
Mastery Quiz 12
Due Tuesday, April 23

This week's mastery quiz has four topics. Everyone should submit topics S9 and S10.. If you have a 4/4 on M3 or M4, you don't need to submit them.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Tuesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 3: Series Convergence

- Major Topic 4: Taylor Series

- Secondary Topic 8: Power Series

Name:

Recitation Section:

M3: Series Convergence

(a) Analyze the convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 + n - 3}{n^2 4^n}$

(b) Analyze the convergence of the series $\displaystyle\sum_{n=2}^{\infty} \frac{\ln(n) + n}{n^2 - 1}$

(c) Analyze the convergence of the series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3n^4 - 1}{n^5 + 1}$.

M4: Taylor Series

(a) Let $f(x) = \cos^2(x)$. Use *the definition of a Taylor series* to find $T_4(x, \pi)$ for this function. (That is, find the terms up through the degree four term.)

(b) Using series we already know, write down a formula for the (infinite) Taylor series for $(1 - 2x)^{-3}$, and then write down the degree-four polynomial explicitly.

(c) If $f(x) = \displaystyle\sum_{n=0}^{\infty} 2^n n^3 (x-2)^n$, compute $\frac{d}{dx} f(x)$ and $\int f(x)\, dx$.
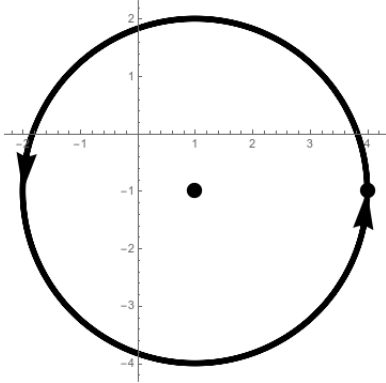
S9: Applications of Taylor Series

(a) Use a degree-three Taylor polynomial to estimate $\sqrt{1.2}$.

(b) Use a Taylor series to compute $\displaystyle\lim_{x\to 0} \frac{xe^{x^3} - x - x^4}{x^7} =$

(c) Using series, compute $\int_0^\pi 2x\cos(x^5)\,dx$.

S10: Parametrization

(a) Find a parametrization for the circle of radius 3 centered at $(1, -1)$, starting at $(4, -1)$ and going **counterclockwise twice** around the circle.

(b) Find a parametrization of the ellipse $x^2/4 + y^2 = 1$. (Hint: what are the x and y intercepts?)

(c) Find a parametric equation for the line tangent to the curve $x = 1 + \sqrt{t}, y = t^3$ at the point $(2, 1)$.