Math 1232: Single-Variable Calculus 2
George Washington University   Spring 2024
Recitation 12

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Problem 1. Let's find the Taylor series of $f(x) = e^x$ centered at $a = 1$.

(a) Compute f', f'', f'''. Find a formula for $f^{(n)}(x)$.

(b) Give a formula for $T_f(x, 1)$.

(c) We want to know if $f(x) = T_f(x, 1)$. Find a formula for $R_k(x, 1)$. Can you show this goes to 0 as k goes to infinity?

(d) We already have another power series for f:

$$T_f(x, 0) = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

You should have a different power series; but can you convince yourself it *should* give the same function? (What happens if you plug $x - 1$ into this series?)

Problem 2. Let's do something silly, and compute the Taylor series of a polynomial.

(a) Let $f(x) = x^3 + 3x^2 + 1$. Find the Taylor series centered at zero. Was that what you expected?

(b) Now find the Taylor series centered at 2. Do you get the same thing? What's useful about this?

Problem 3. Back in section 2 we talked about the bell curve function $p(x) = e^{-x^2}$. (Technically we should be talking about $\frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ but that's annoying and doesn't change the details enough to be interesting.)

(a) Find a power series for $p(x)$ centered at zero. (This should not require any real calculations.)

(b) Find an antiderivative for $p(x)$, using power series.

(c) Write down a series that computes $\int_0^1 p(x)\,dx$.

(d) Add up the first three or four terms of this series. What do you get? Can you estimate the error in this calculation?

Problem 4. In class we worked out a Taylor series for $g(x) = \ln(x)$ centered at $a = 1$:

$$T_g(x, 1) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}(x-1)^n.$$

But is this actually equal to $g(x)$?

(a) Write down a formula for $R_k(x, 1)$.

(b) Compute $T_5(2, 1)$. Can you estimate the error?

(c) Compute $T_5(1.5, 1)$. Can you estimate the error?

(d) Compute $T_5(0, 1)$. Can you estimate the error here?

(e) What would you need to assume to show this goes to zero as k goes to infinity? Does that makes sense?