Math 1232 Spring 2024
Single-Variable Calculus 2 Section 12
Mastery Quiz 2
Due Tuesday, January 30

This week's mastery quiz has two topics. Everyone should submit work on topic M1. If you got a 2/2 on S1 last week, you don't need to submit it again; if you got a 1/2 or 0/2, you should try again after consulting the solutions to last week's quiz.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Tuesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 1: Calculus of Transcendental Functions
- Secondary Topic 1: Invertible Functions

Name:

Recitation Section:

M1: Calculus of Invertible Functions

(a) Compute $\dfrac{d}{dx}x^{\ln(x)}$.

Solution: The simplest approach is to use logarithmic differentiation.

$$\begin{aligned} y &= x^{\ln(x)} \\ \ln(y) &= \ln(x)\ln(x) = \ln(x)^2 \\ \frac{y'}{y} &= 2\ln(x)\frac{1}{x} \\ y &= \frac{2\ln(x)}{x}y = \frac{2\ln(x)x^{\ln(x)}}{x}. \end{aligned}$$

Alternatively, we could compute

$$\begin{aligned} \frac{d}{dx}x^{\ln(x)} &= \frac{d}{dx}\left(e^{\ln(x)}\right)^{\ln(x)} = \frac{d}{dx}e^{\ln(x)^2} \\ &= e^{\ln(x)^2} \cdot 2\ln(x)\frac{1}{x} = \frac{2\ln(x)x^{\ln(x)}}{x}. \end{aligned}$$

(b) $\displaystyle\int \frac{e^{3y}}{e^{3y}+5}\,dy =$

Solution: Set $u = e^{3y} + 5$ so $du = 3e^{3y}\,dy$, and we have

$$\int \frac{e^{3y}}{e^{3y}+5}\,dy = \int \frac{du}{3u} = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|e^{3y}+5| + C.$$

(c) Find an equation for the tangent line to the curve $y = \ln(2x^2 - 3x - 1)$ at the point $(2, 0)$.

Solution: We have $y' = \frac{1}{2x^2-3x-1}(4x-3)$ and thus $y'(2) = \frac{5}{1} = 5$. So the equation of the tangent line is

$$y - 0 = 5(x - 2).$$

S1: Invertible Functions

(a) Let $h(x) = \sqrt{x^5 + x + 2}$. Compute $(h^{-1})'(6)$.

Solution: By the Inverse Function Theorem, we know that

$$(h^{-1})'(2) = \frac{1}{h'(h^{-1}(6))}.$$

Guess and check shows that $h(2) = 6$ so $h^{-1}(6) = 2$. And we know that

$$h'(x) = \frac{1}{2}(x^5 + x + 2)^{-1/2}(5x^4 + 1)$$

and thus

$$h'(2) = \frac{1}{12}(81)$$

Thus

$$(h^{-1})'(6) = \frac{12}{81}.$$

(b) Find a formula for the inverse of $g(x) = 2 + \sqrt{3 - x}$.

Solution:

$$\begin{aligned} y &= 2 + \sqrt{3 - x} \\ y - 2 &= \sqrt{3 - x} \\ (y - 2)^2 &= 3 - x \\ x &= 3 - (y - 2)^2 \end{aligned}$$

so $f^{-1}(y) = 3 - (y - 2)^2$. (You can use whichever variable you like in your formula.)

(c) Compute $e^{5\ln(3) - 2\ln(4)}$. (Give an exact answer with no decimals.)

Solution:

$$e^{5\ln(3) - 2\ln(4)} = (e^{\ln(3)})^5/(e^{\ln(4)})^2 \qquad = \frac{3^5}{4^2} = \frac{243}{16}.$$