Math 1232 Spring 2024
Single-Variable Calculus 2 Section 12
Mastery Quiz 3
Due Tuesday, February 6

This week's mastery quiz has two topics. Everyone should submit work on topic M1 (even if you got a 2 last week; your best two scores count). Everyone should also submit topic S2, which is new.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Tuesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 1: Calculus of Transcendental Functions
- Secondary Topic 2: L'Hospital's Rule

Name:

Recitation Section:

M1: Calculus of Invertible Functions

(a) (Note this is a definite integral)
$$\int_0^2 \frac{e^{2x}}{e^{4x}+1}\,dx =$$

Solution: We can take $u = e^{2x}$ so $du = 2e^{2x}\,dx$ and

$$\int_0^2 \frac{e^{2x}}{e^{4x}+1}\,dx = \int_1^{e^4} \frac{1}{2}\frac{1}{u^2+1}\,du = \frac{1}{2}\arctan(u)\Big|_1^{e^4}$$
$$= \frac{1}{2}\arctan(e^4) - \frac{1}{2}\arctan(1) = \frac{1}{2}\arctan(e^4) - \frac{\pi}{8}.$$

(b) Compute $\frac{d}{dx}\left(\sqrt{x+1}\right)^x$

Solution:

$$y = \sqrt{x+1}^x$$
$$ln|y| = x\ln(\sqrt{x+1}) = \frac{1}{2}x\ln(x+1)$$
$$y'/y = \frac{1}{2}\left(\ln(x+1) + \frac{x}{x+1}\right)$$
$$y' = \frac{1}{2}\sqrt{x+1}^x\left(\ln(x+1) + \frac{x}{x+1}\right)$$

(c) $\frac{d}{dx}\frac{1}{\arcsin(x^2)} =$

Solution:
$$\frac{d}{dx}\frac{1}{\arcsin(x^2)} = \frac{-1}{\arcsin(x^2)^2}\frac{1}{\sqrt{1-x^4}}\cdot 2x.$$

S2: L'Hospital's Rule

(a) $\lim_{x\to 0}\frac{x^3-x^2}{x+\sin(x)} =$

Solution: The limits of the top and bottom are both zero, so we can use L'Hospital's Rule:

$$\lim_{x\to 0}\frac{x^3-x^2{}^{\nearrow 0}}{x+\sin(x)_{\searrow 0}} \overset{\text{L'H}}{=} \lim_{x\to 0}\frac{3x^2-2x^{\nearrow 0}}{1+\cos(x)_{\searrow 2}} = \frac{0}{2} = 0.$$

Note we *cannot* use L'Hospital's rule a second time, because we don't have an indeterminate form.

(b) $\lim_{x\to+\infty} \frac{\arctan(x)}{\arctan(x)+1} =$

Solution: $\lim_{x\to+\infty} \arctan(x) = \pi/2$, so this limit is $\frac{\pi/2}{\pi/2+1} \approx .611$.

Note: you cannot use L'Hospital's rule here! If you tried, you would get

$$\lim_{x\to+\infty} \frac{1/(x^2+1)}{1/(x^2+1)} = \lim_{x\to+\infty} 1 = 1$$

but that is not in fact the limit.

(c) $\lim_{x\to\infty} x^{\frac{3}{2+\ln(x)}} =$

Solution:

$$\ln y = \frac{3}{2+\ln(x)} \ln(x)$$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{3\ln(x)^{\nearrow\infty}}{2+\ln(x)_{\searrow\infty}}$$

$$\stackrel{\text{L'H}}{=} \lim_{x\to\infty} \frac{3/x}{1/x} = 3$$

and thus

$$\lim_{x\to\infty} y = e^3.$$