Math 1232: Single-Variable Calculus 2
George Washington University Spring 2024
Recitation 5

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Problem 1. We want to find $\displaystyle\int \frac{x^5 + x - 1}{x^3 + 1}\, dx$.

(a) What's the first tool we need to apply here? (Hint: not partial fractions!)

(b) Once we get it in a more manageable form, things should simplify out nicely. What is the final integral?

Problem 2. We've looked briefly at the integral $\displaystyle\int \frac{1}{1 + e^x}\, dx$. Let's try it again with our new tools.

(a) Try the substitution $u = e^x$. What do you get? What tools can apply to the result?

(b) Do a partial fractions decomposition to get the integral.

Problem 3 (Bonus)**.** Let's see if we can work out the integral of secant! This isn't at all obvious.

(a) We want $\int \sec(x)\, dx = \int \frac{1}{\cos(x)}\, dx$. Since this is a fraction, we can multiply the top and bottom through by $\cos(x)$. This makes the expression more complicated, but it does allow us to use a trig identity. What do we get?

(b) Now we can do a u substitution. What u substitution seems reasonable? Does it help us at all?

(c) Now we can use partial fractions to finish the problem off. We wind up with an awkward answer, but an answer.

(d) The most common formula for the integral of $\sec(x)$ is $\ln|\sec(x) + \tan(x)| + C$. Is that the same as what you got? (Hint: use logarithm laws and multiplication by the conjugate.)

Problem 4 (Bonus)**.** What if we want to find $\int \frac{x^4 + 6x^3 + 4x^2 + 8x + 11}{(x-1)^2(2x+1)(x^2+4x+5)}\, dx$?

Problem 5. We want to find the cross-sectional area of a two-meter-long airplane wing. We measure its width every 20 centimeters, and get: 5.8, 20.3, 26.7, 29.0, 27.6, 27.3, 23.8, 20.5, 15.1, 8.7, 2.8. Use the trapezoidal rule and Simpson's rule to estimate the area of the wing.

Problem 6. Consider the function $f(x) = x^2 + 1$.

(a) Use the trapezoid rule with six intervals to estimate $\int_{-4}^{2} f(x)\, dx$.

(b) Use the midpoint rule with six intervals to estimate $\int_{-4}^{2} f(x)\, dx$.

(c) Use Simpson's rule with six intervals to estimate $\int_{-4}^{2} f(x)\, dx$.

(d) Which of these do you expect to be most accurate? Which do you expect to be least accurate?

(e) Compute $\int_{-4}^{2} f(x)\, dx$. What do you find? Why?

Problem 7. Let $g(x) = e^{-x^2}$, and suppose we want to compute $\int_{-1}^{2} e^{-x^2}\, dx$, and get the answer correct to two decimal places.

(a) We can compute that $g''(x)$ varies between -2 and $.9$ when x is in $[-1, 2]$. What value should we take for K?

(b) How many subintervals should we use to get the answer correct to within two decimal places using the trapezoid rule?

(c) How many subintervals should we use to get the answer correct to within two decimal places using the midpoint rule?

(d) We can compute that $g''''(x)$ varies between -8 and 12. What value should we take for L?

(e) How many subintervals should we use to get the answer correct to within two decimal places using Simpson's rule?