Math 1232: Single-Variable Calculus 2
George Washington University Spring 2023
Recitation 6

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Problem 1. We want to compute $\int_0^2 \frac{1}{\sqrt[3]{x-1}}\,dx$.

(a) Can you compute an antiderivative? Can you evaluate it at 0 and 2?

(b) Did part (a) finish the problem? Sketch a picture of the graph. What should we be concerned about?

(c) Carefully set up a computation that will find $\int_0^2 \frac{1}{\sqrt[3]{x-1}}\,dx$. (Hint: It should have two limit operations in it.)

(d) What did we learn from this that we didn't learn from (a)?

Problem 2. We want to figure out if $\int_0^{+\infty} e^{-x^2}\,dx$ converges—that is, if it's finite or infinite.

(a) If we can find an antiderivative, we can just compute the improper integral directly. Why doesn't that work?

(b) Since we can't integrate this directly we might want to use a comparison test. We need to find an easy-to-integrate function that's larger than e^{-x^2}. Find a function $f(x)$ that makes $f(x)e^{-x^2}$ easy to integrate.

(c) If $f(x) \geq 1$, then we can just integrate $f(x)e^{-x^2}$. Is it?

(d) This is where we can pull in a trick. Is there some a where $f(x) > 1$ when $x > a$? (You may need to adjust your $f(x)$ here, especially the sign. It's fine as long as you can still integrate it.)

(e) We know $\int_a^{+\infty} e^{-x^2}\,dx \leq \int_a^{+\infty} f(x)e^{-x^2}\,dx$. Compute the new improper integral; is it finite?

(f) Now we just have to deal with $\int_0^a e^{-x^2}\,dx$. We can't do that integral exactly, but that's fine: you should be able to tell whether it's finite or not without doing any calculations. How?

(g) Does $\int_0^{+\infty} e^{-x^2}\,dx$ converge?

Problem 3. Let $f(x) = x^2$. Let's find the arc length between $x = 0$ and $x = 4$.

(a) This makes a very reasonable shape. What does the graph look like?

(b) Set up an integral to compute this arc length. You need to think about the variable of integration, the bounds, and the actual function to integrate.

(c) What techniques should we use to compute this integral? Where do we get stuck?

(d) Is there another way we could have set it up?

(e) Is that integral any easier?

Problem 4. Let $f(x) = \sqrt[3]{3x}$. Take the portion of the graph where $0 \leq y \leq 2$ and rotate it around the y axis.

(a) Try to sketch a picture of what this will look like.

(b) Set up an integral to find the surface area. Again, think about the variable of integration, the bounds, and the function. Do you have multiple choices here or just one?

(c) Can you compute that integral?