Math 1232: Single-Variable Calculus 2
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Recitation 8

Jay Daigle

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Problem 1. Let $(a_n) = \left(-6, 4, \frac{-8}{3}, \frac{16}{9}, \frac{-32}{27}, \dots\right)$.

(a) Find a closed-form formula for a_n.

(b) Is there a real function f so that $f(n) = a_n$?

(c) What is $\lim_{n\to\infty} a_n$? Why?

Problem 2 (Factorials). (a) What is 4!? What is $\frac{4!}{3!}$?

(b) What is $\frac{5!}{4!}$? What is $\frac{5!}{3!}$?

(c) Can you figure out what $\frac{202!}{200!}$ is?

Problem 3. (a) Compute $\displaystyle\lim_{n\to\infty} \frac{n}{n!}$. Justify your answer.

(b) Compute $\lim_{n\to\infty} \frac{e^n}{n!}$.

(c) Now compute $\lim_{n\to\infty} \frac{n^k}{n!}$, where $k > 0$ is an integer.

Problem 4. Consider the sequence $(a_n) = (\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots)$.

(a) We don't have a closed-form formula for this sequence, but we can still say things about it. What happens if we square each element of the sequence, and then divide by 2?

(b) We want to find the limit of this sequence. Half of this is easy: *if* the sequence converges, we can use a trick to find the limit.

Suppose $\lim_{n\to\infty} a_n = L$. What can you say about $L^n/2$?

(c) Can you figure out what L is, if the limit exists?

(d) That all relied on the idea that the limit existed. We want to use completeness to prove this. We need to show this sequence is increasing and bounded above.

If $0 \leq x \leq 2$, explain why $x \leq \sqrt{2x}$.

(e) If $0 \leq x \leq 2$, explain why $\sqrt{2x} \leq 2$.

(f) How does this prove the limit exists?

Problem 5. The discrete equivalent of a derivative is a *difference quotient*. Given a function $f(n)$ defined on positive integers, we can define $\Delta f(n) = f(n+1) - f(n)$.

(a) Does that look like a derivative? What pieces are missing, and why?

(b) If $f(n) = n^2$, compute $\Delta f(n)$. Compute $f'(n)$. How are they related?

(c) If $g(n) = \frac{1}{n}$, compute $\Delta g(n)$. Compute $g'(n)$. How are they related?