Math 1232: Single-Variable Calculus 2

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Recitation 9

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Problem 1. Write out the first five terms of:

(a) $\sum_{k=1}^{\infty} \frac{(-2)^{k+1}}{3k}$

(b) $\sum_{k=1}^{\infty} \frac{k+1}{k!}$

(c) $\sum_{k=3}^{\infty} \frac{k+3}{k^2-k-2}$

Problem 2. Write in series/summation notation:

(a) $1 + \frac{2}{3} + \frac{3}{5} + \frac{4}{7} + \dots$

(b) $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} + \dots$

(c) $2 + 7 + 14 + 23 + 34 + \dots$

Problem 3. (a) Use a telescoping series argument to write down a formula for $\sum_{k=1}^{n} \frac{1}{k^2+3k+2}$.

(b) Compute $\sum_{k=1}^{\infty} \frac{1}{k^2+3k+2}$.

(c) Use a telescoping series argument to write down a formula for $\sum_{k=1}^{n} \frac{2}{k^2+2k}$.

(d) Compute $\sum_{k=1}^{\infty} \frac{2}{k^2+2k}$.

(e) Use a telescoping series argument to write down a formula for $\sum_{k=1}^{n} \ln\left(\frac{k+1}{k+3}\right)$.

(f) Compute $\sum_{k=1}^{\infty} \ln\left(\frac{k+1}{k+3}\right)$.

Problem 4 (Geometric Series)**.** Compute:

(a) $\sum_{k=1}^{\infty} \frac{2^k}{3^k}$

(b) $\sum_{k=2}^{\infty} \frac{(-5)^{k+2}}{2^{3k}}$

(c) $\frac{5}{2} + \frac{5}{4} + \frac{5}{8} + \frac{5}{16} + \dots$

(d) $\frac{-2}{3} + \frac{8}{9} + \frac{-32}{27} + \dots$

(e) $\frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81} + \dots$

Problem 5 (Infinite Decimals)**.** We want to find a rational representation of the infinite decimal $0.\overline{47}$. That is, we want to write $0.\overline{47} = \frac{p}{q}$ for integers p, q.

(a) First, what happens if we multiply $0.\overline{47}$ by 100?

(b) Using part (a), what can you tell about $(99) \cdot 0.\overline{47}$?

(c) Give a rational representation of $0.\overline{47}$.

(d) Now let's take a different approach. Write $0.\overline{47}$ as an infinite series.

(e) What kind of series is this? Can you use that fact to find a rational representation of $0.\overline{47}$?

(f) Now use the same logic to find a rational representation of $2.\overline{63}$.