Syllabus and Mathematical Reasoning

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What are we doing here?

- Math isn't just numbers and equations
  - Identify our assumptions
  - Describe them explicitly
  - Understand implications
- Mathematical ideas can explain political systems
  - How do we decide who wins elections?
  - How do we allocate political power?
  - How do we negotiate successfully?
- Approach questions like a mathematician
  - Articulate our thoughts clearly
  - Understand our own priorities
  - Converse with and persuade effectively

Boring Logistics

Textbook

The Mathematics of Politics, Second Edition

by E. Arthur Robinson and Daniel H. Ullman

https://wrlc-gwu.primo.exlibrisgroup.com/permalink/01WRLC_GWA/1j51gk4/alma9918591806220410 7

Course Web Page

https://jaydaigle.net/politics/

Linked from Blackboard

Contacting me

Me

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Office Hours

Office: Phillips 720E

Office hours:

Tuesday 12:00–2:00 PM

Thursday 11:00 AM–2:00 PM

Unit 1: Voting

Aug 25	Mathematical Reasoning	Aug 27	Voting Systems
~~Sept 1~~	**No Class for Labor Day**	Sept 3	Two-Candidate Elections
Sept 8	Criteria for Voting Systems, **Quiz**	Sept 10	Multi-Candidate Criteria
Sept 15	Theorems on Voting System Criteria	Sept 17	Evaluating Voting Systems I
Sept 22	Evaluating Voting Systems II, **Quiz**	Sept 24	Arrow's Impossibility Theorem
Sept 29	**Midterm 1**		

Unit 2: Apportionment

		Oct 1	The Problem of Apportionment
Oct 6	Hamilton's Method	Oct 8	Jefferson's Method
Oct 13	Divisor Methods, **Quiz**	Oct 15	More on Divisor Methods
Oct 20	Evaluating Apportionment Methods	Oct 22	Criteria and Impossibility, **Quiz**
Oct 27	Balinski and Young Apportiomnent	Oct 29	**Midterm 2**

Unit 3: Conflict and Game Theory

Nov 3	Zero-Sum Games	Nov 5	Strategies and Outcomes
Nov 10	Probability and Randomness	Nov 12	Expected Value and Strategies, **Quiz**
Nov 17	Solving Zero-Sum Games	Nov 19	Conflict and Cooperation
~~Nov 24~~	**Thanskgiving**	~~Nov 26~~	**Thanskgiving**
Dec 1	Non-Zero-Sum Games	Dec 3	Nash Equilibria, **Quiz**
Dec 8	Some Important Games		

Course Structure

Grade Distribution

- Homework: 25% (drop two lowest)
- Tests: 15% each (30% total)
- Quizzes: 4% each (20% after dropping lowest)
- Final Exam: 25%

Homework

- Written or typed
- Due every week, usually in class on Wednesday
- Must be your own work
- Two lowest scores are dropped
- For your benefit, so you can practice

Quizzes and Tests

Quiz schedule

- Six quizzes, two on each unit
- Timed, in-class
- Usually about 20 minutes
- Check your understanding of the material as we go

Test Schedule

- Test 1 on Voting: Monday, September 29
- Test 2 on Apportionment: Wednesday, October 29
- Final exam: tentatively Monday December 15 at 12:40 PM

Do not book travel before finals are over!

Mathematical reasoning

1. Definitions
2. Proofs

Definitions

- Precise, not fuzzy
- Taken literally
- Not normal English meanings!
- Similar to legal reasoning

Is a bee a fish?

Each of these statutes provides that covered species include "native species or subspecies of a bird, mammal, fish, amphibian, reptile, or plant[.]" This portion of the code, however, does not elaborate on what qualifies as a bird, mammal, fish, and so forth. Based only on the qualified species listed above, bees and other land-dwelling invertebrates would not receive protection under the law. The court looked elsewhere in the Fish and Game Code for definitions to help clarify whether bees may qualify for protection under CESA.

Is a bee a fish?

Importantly, the section 45 of the code defines “fish” as “a wild fish, mollusk, crustacean, invertebrate, amphibian, or part, spawn, or ovum of any of those animals.” (Emphasis added). According to the court, the term “invertebrate” under the definition of fish includes both aquatic and terrestrial invertebrates, such as bees.

Definition of a function

Definition

A function is a rule that assigns exactly one output to every valid input. We call the set of valid inputs the domain of the function, and the set of possible outputs the codomain or sometimes the range. A function must be deterministic, in that given the same input it will always yield the same output.

Definition (Unnecessarily technical definition)

Let A, B be sets. We define a function $f : A \to B$ to be a set of ordered pairs $\{(a, b) : a \in A, b \in B\}$ such that for each a in A there is exactly one pair whose first element is a. We call A the domain and B the codomain of f.

Proofs

Types of theorems

- "Theorem" for a major, important result
- "Proposition" for a less important result that we still care about
- "Lemma" for annoying technical results we mainly want in order to prove something else we actually care about
- "Corollary" for something that follows immediately from something we've already proven.

Theorems are universal

- Need universal arguments to prove something is always true
- Need one counterexample to prove something is not always true
- (Need universal arguments to prove something is never true)

Example

- All swans are white
- No swans are red
- No black swan is white

Hypotheses: When Things Break

A theorem tells you when something can break

Hypotheses are conditions that force a result to be true.

Example

If we decrease revenue and increase spending, the deficit will increase.

Example

A voting method that is anonymous and neutral cannot be decisive.

Theorems tell us about tradeoffs we can't avoid.