Conflict and Cooperation

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December 1, 2025

Escaping the conflict

- This is a framework for conflict with another player
- Zero sum: no room for cooperation
- Can win or lose a war, but can't avoid it
- Can we model the possibility of cooperation?

Bimatrix Games

Definition

- A bimatrix is a rectangular array in which each cell has an ordered pair of numbers.
- Given a row i and a column j, we write $u_{i,j}$ for the first number in this cell, and $v_{i,j}$ for the second number.

Example

0, 3	-1, 2	0, -2
1, 0	0, 1	0, 0
2, -1	-1, -6	0, 3

- $u_{1,2} = -1$
- $v_{1,2} = 2$

Bimatrix Games

Definition (Bimatrix game)

In a bimatrix game:

- Player one picks a row i
- Player two picks a column j
- Player one gets the payoff $u_{i,j}$
  - Wants $u_{i,j}$ to be big
  - Doesn't care about $v_{i,j}$ at all
- Player two gets the payoff $v_{i,j}$
  - Wants $v_{i,j}$ to be *big*
  - Doesn't care about $u_{i,j}$ at all

Bimatrix Games

Example

0, 3	-1, 2	0, -2
1, 0	0, 1	0, 0
2, -1	-1, -6	0, 3

- Pick R3 and C3
  - Row wins 0
  - Column wins 3
- Picks R3 and C2
  - Row loses 1
  - Column loses 6.

- (3,3) better for both players than (3,2). Not zero-sum!
- Any zero-sum game can be represented as a bimatrix game
  - Second number happens to be opposite of first
- But many bimatrix games are not zero-sum.

Guarantees and Saddle Points

Definition

- The guarantee of a strategy is the worst payoff it can produce
  - The guarantee of Row k is the smallest $u_{k,j}$ for any j
  - The guarantee of Column ℓ is the *smallest* $v_{i,\ell}$ for any i
  - Both Row and Column want bigger numbers!
  - They're just looking at *different* numbers.
- The prudent strategy is the strategy with the largest guarantee
- The prudent strategy method tells a player to choose their prudent strategy.

Guarantees and Saddle Points

- Can make a min-max diagram as before

Example

0, 3	-1, 2	0, -2	-1
1, 0	0, 1	0, 0	0
2, -1	-1, -6	0, 3	-1
-1	-6	-2	

- Row's guarantee: 0
- Column's guarantee: -1
- Doubly prudent:
  - Row wins 1
  - Columns breaks even
  - Both better than guarantees

Guarantees and Saddle Points

Definition

- The best response to an opponent strategy is the strategy that gets the best outcome if your opponent plays that strategy
- Can make a flow diagram

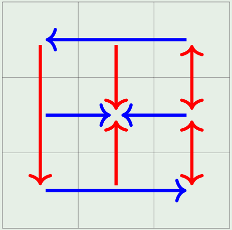
Definition

- A saddle point is an outcome where each player's strategy is a best response to the opponent's strategy.
- Saddle points happen in cells where every flow diagram arrow points inwards.

Guarantees and Saddle Points

Example

0, 3	-1, 2	0, -2
1, 0	0, 1	0, 0
2, -1	-1, -6	0, 3

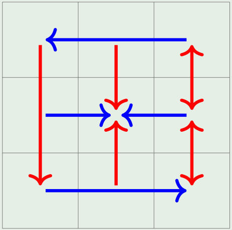


- Suppose Row chooses R1
- Column will pick C1
- Row wants R3
- Column wants C3

- Row stays put
- (3,3) is a saddle point
- Payoff $(0, 3)$

Example

0, 3	-1, 2	0, -2
1, 0	0, 1	0, 0
2, -1	-1, -6	0, 3



- Suppose Column chooses C2
- Row will pick R2
- Column stays put
- (2,2) is a saddle point
- Payoffs $(0, 1)$

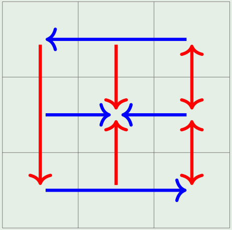
Discussion Question

- Is one saddle point better than the other?

Guarantees and Saddle Points

Example

0, 3	-1, 2	0, -2
1, 0	0, 1	0, 0
2, -1	-1, -6	0, 3



- In zero-sum games, all saddle points are equivalent
- In non-zero sum games, that's not true!
- Everyone thinks (3,3) is at least as good as (2,2)
- Important question: how to coordinate?

Cooperation and Competition

0, 3	-1, 2	0, -2
1, 0	0, 1	0, 0
2, -1	-1, -6	0, 3

3rd, 1st	4th, 2nd	3rd, 6th
2nd, 4th	3rd, 3rd	3rd, 4th
1st, 5th	4th, 7th	3rd, 1st

- Row payoffs: 2, 1, 0, -1
- Column payoffs: 3, 2, 1, 0, -1, -2, -6
- Can rank each outcome

Cooperation and Competition

Definition

- A coordination game is a game where both players' preference orders are exactly the same.
- A strictly competitive game is one where the preference orders are exactly opposite.
- A mixed motive game is any other game, which combines coordination and competition.

Proposition

Any zero-sum or constant-sum game is strictly competitive.

- Most interesting games are the mixed motive games.
- But not all!

Meet in New York

Example

- You and your friend want to meet in New York city at noon.
- You forgot to agree on where to meet.

Discussion Question

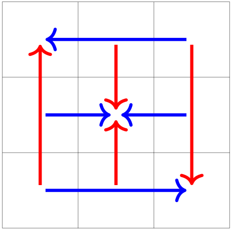
Where would you go?

- Some ideas: Times Square, Grand Central, Empire State

	Times Square	Grand Central	Empire State
Times Square	1, 1	0, 0	0, 0
Grand Central	0, 0	1, 1	0, 0
Empire State	0, 0	0, 0	1, 1

Meet in New York

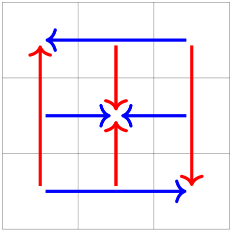
	TS	GC	ES
TS	1, 1	0, 0	0, 0
GC	0, 0	1, 1	0, 0
ES	0, 0	0, 0	1, 1



- Is this cooperative, competitive, or mixed?
    - Purely cooperative
- Are there any saddle points?
    - Three saddle points: TS,TS; GC,GC; ES,ES
- What strategy should you play?
    - That's a hard question!

Meet in New York

	TS	GC	ES
TS	1, 1	0, 0	0, 0
GC	0, 0	1, 1	0, 0
ES	0, 0	0, 0	1, 1

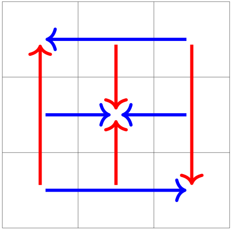


Definition

- Two outcomes are equivalent if both players agree they are equally good
  - (i, j) is equivalent to (k, ℓ) if $u_{i,j} = u_{k,\ell}$ and $v_{i,j} = v_{k,\ell}$
- The set of outcomes that are all equivalent to each other is an equivalence class.

Meet in New York

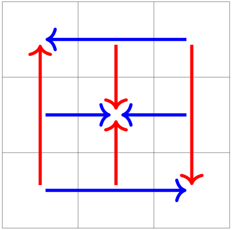
	TS	GC	ES
TS	1, 1	0, 0	0, 0
GC	0, 0	1, 1	0, 0
ES	0, 0	0, 0	1, 1



- Thomas Schelling (1921-2016)
- Economist at Yale, Harvard, and UMD
- Asked his class every year where they would meet
- Most common answer: Grand Central Station
- Called a focal point or Schelling point

Meet in New York

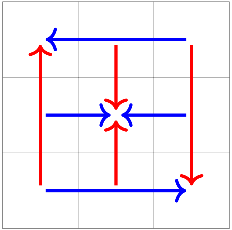
	TS	GC	ES
TS	1, 1	0, 0	0, 0
GC	0, 0	1, 1	0, 0
ES	0, 0	0, 0	1, 1



- What should you play?
    - Grand Central
- Why?
    - You know everyone else just heard that suggestion!

Meet in New York

	TS	GC	ES
TS	1, 1	0, 0	0, 0
GC	0, 0	1, 1	0, 0
ES	0, 0	0, 0	1, 1



Discussion Question

Should the payoffs really be equal?

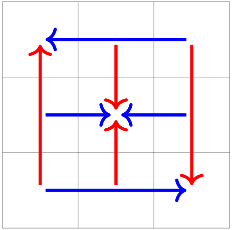
- Can you actually find each other in Times Square?
- From DC, maybe makes more sense to meet at Moynihan Train Station.

Interview with Thomas Schelling, Econ Focus Spring 2005

When I first asked that question, way back in the 1950s, I was teaching at Yale. A lot of the people to whom I sent the questionnaire were students, and a large share of them responded: under the clock at the information desk at Grand Central Station. That was because in the 1950s most of the male students in New England were at men's colleges and most of the female students were at women's colleges. So if you had a date, you needed a place to meet, and instead of meeting in, say, New Haven, you would meet in New York. And, of course, all trains went to Grand Central Station, so you would meet at the information desk. Now when I try it on students, they almost never give that response.

Meet in New York

	TS	GC	ES
TS	1, 1	0, 0	0, 0
GC	0, 0	2, 2	0, 0
ES	0, 0	0, 0	1, 1



- Still three saddle points
- What would you pick?

Meet in New York

- Easy choice if there's a unique doubly-primary outcome
- Meet in New York doesn't have that—not unique
- Sometimes the same idea works without actual uniqueness.

	TS	GC	ES
TS	1, 1	0, 0	0, 0
GC	0, 0	1, 1	0, 0
ES	0, 0	0, 0	1, 1

	A	B	C
A	2, 2	0, 0	2, 2
B	0, 0	1, 1	0, 0
C	2, 2	0, 0	2, 2

Meet in New York

Discussion Question

When does this come up?

Example

- Meeting your friends
- Social media networks (or chat apps)
- Windows or Apple?
- Keyboard layout
- Drive on the right or the left?
- National borders

Interchangeability

	TS	GC	ES
TS	1, 1	0, 0	0, 0
GC	0, 0	1, 1	0, 0
ES	0, 0	0, 0	1, 1

	A	B	C
A	2, 2	0, 0	2, 2
B	0, 0	1, 1	0, 0
C	2, 2	0, 0	2, 2

Definition

We say a set of outcomes S is interchangeable if whenever both players aim at any outcome in S, the result will be in S.

Proposition

- *In a 2P zero-sum game, all saddle points are interchangeable.*
- *In a non-zero-sum game, they may not be interchangeable.*

Games as Metaphors

- Rarely play bimatrix games in daily life
- Useful as a framework for thinking about strategic decisions
- Want to leave you with several "standard" games that are useful metaphors for situations we encounter
- Think about the math of the game, but also when you see similar situations in your life

A disclaimer

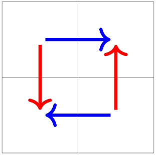
- The "standard" presentations are mostly from the 1950s.
- You can tell.
- I'll mostly present them as historically given, and then talk about more modern versions.

Battle of the Sexes

Example

- R. Duncan Luce and Howard Raiffa (1957)
- Row and Column want to go on a date
- They have two options: boxing match or ballet recital
- Row prefers ballet and Column prefers boxing
- But more importantly they want to go *together*.

	Boxing	Ballet
Ballet	0, 0	10, 5
Boxing	5, 10	-5, -5



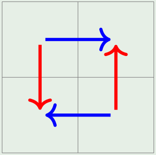
Discussion Question

What should each player do?

Battle of the Sexes

Example

	Boxing	Ballet
Ballet	0, 0	10, 5
Boxing	5, 10	-5, -5

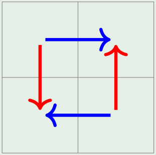


- Two saddle points: both go to boxing or both go to ballet
- Also a mixed Nash equilibrium:
  - Row goes boxing 1/4 the time
  - Column goes to ballet 1/4 the time.
  - Expected value for each player is 2.5.
- All of these equilibria seem bad
  - The first two are persistently unfair
  - The third is worse for both players than *either* saddle point.

Battle of the Sexes

Example

	Boxing	Ballet
Ballet	0, 0	10, 5
Boxing	5, 10	-5, -5

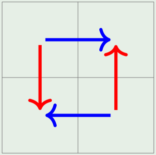


- "Reasonable" solution: compromise.
  - Take turns or flip a coin
  - Either way, expected value is 7.5.
- What if you can't coordinate?

Battle of the Sexes

Example

	Boxing	Ballet
Ballet	0, 0	10, 5
Boxing	5, 10	-5, -5



- Or what if you're an asshole?
  - Loudly announce you're going to the ballet
  - Column will be better off yielding
  - Requires credible precommitment

Battle of the Sexes

Key Questions

- How to distribute benefits fairly
- How to get more of the pie without shrinking the pie

Example

- Where to go on a date
- Pricing a deal
- Signing a treaty
- Congressional bills