Famous Games that Make Useful Metaphors

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Games as Metaphors

- Rarely play bimatrix games in daily life

- Useful as a framework for thinking about strategic decisions

- Want to leave you with several "standard" games that are useful metaphors for situations we encounter

- Think about the math of the game, but also when you see similar situations in your life
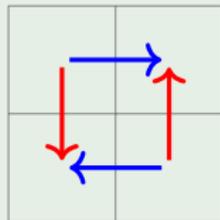
A disclaimer

- The "standard" presentations are mostly from the 1950s.

- You can tell.

- I'll mostly present them as historically given, and then talk about more modern versions.

Battle of the Sexes

Example

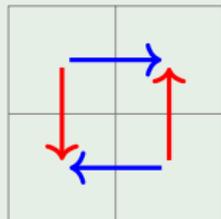|	Boxing	Ballet
Ballet	0, 0	10, 5
Boxing	5, 10	-5, -5

- Two saddle points: both go to boxing or both go to ballet
- Also a mixed Nash equilibrium:
  - Row goes boxing $1/4$ the time
  - Column goes to ballet $1/4$ the time.
  - Expected value for each player is 2.5.
- All of these equilibria seem bad
  - The first two are persistently unfair
  - The third is worse for both players than *either* saddle point.

Example

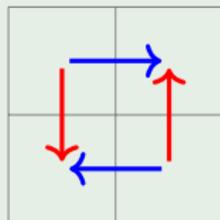|	Boxing	Ballet
Ballet	0, 0	10, 5
Boxing	5, 10	-5, -5

- "Reasonable" solution: compromise.
  - Take turns or flip a coin
  - Either way, expected value is 7.5.
- What if you can't coordinate?

Example

	Boxing	Ballet
Ballet	0, 0	10, 5
Boxing	5, 10	-5, -5

- Or what if you're an asshole?
  - Loudly announce you're going to the ballet
  - Column will be better off yielding
  - Requires credible precommitment

Battle of the Sexes

Key Questions

- How to distribute benefits fairly

- How to get more of the pie without shrinking the pie
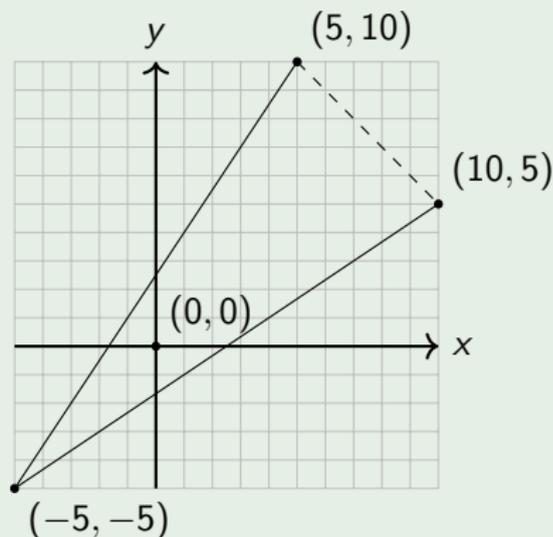
Example

- Where to go on a date

- Pricing a deal

- Signing a treaty

- Congressional bills
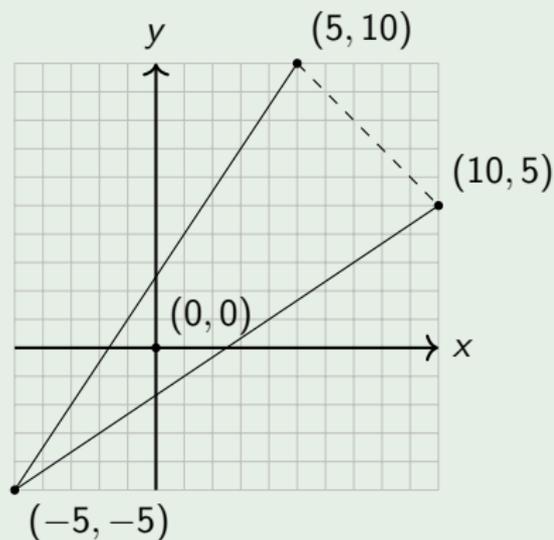
The Payoff Polygon

Definition

- View each pair of payoffs as a point on the plane
- Plot all these points
- Draw the smallest (convex) polygon that contains them
- We call this the payoff polygon

Example (Battle of the Sexes)

The Payoff Polygon

Example (Battle of the Sexes)

- Row wants to move right
- Column wants to move up
- The top-right boundary is Pareto optimal or Pareto efficient
  - Any line segment angled down-right
- Any other point in the polygon is attainable (with mixed strategies), but Pareto inferior

Chicken

Example

- Two drivers drive straight at each other

- If neither swerves, they crash and injure themselves

- But if one swerves, he's a "chicken", which is embarrassing.

	Swerve	Don't
Swerve	0,0	-5, 10
Don't	10, -5	-10, -10
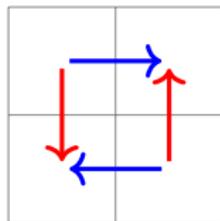
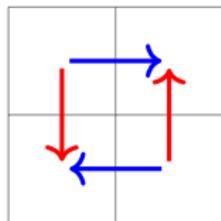### Discussion Question

- What should each player do?

- How is this different from "Battle of the Sexes"?

	Swerve	Don't
Swerve	0,0	-5, 10
Don't	10, -5	-10, -10

- Two saddle points
- Mixed Nash equilibrium: swerve $1/3$ of the time
  - Expected value is $-10/3$
  - Worse than your two good outcomes, but better than either bad outcome.

Chicken versus Battle of the Sexes

	Boxing	Ballet
Ballet	0, 0	10, 5
Boxing	5, 10	-5, -5

	Swerve	Don't
Swerve	0,0	-5, 10
Don't	10, -5	-10, -10

	Boxing	Ballet
Ballet	3rd, 3rd	1st, 2nd
Boxing	2nd, 1st	4th, 4th

	Swerve	Don't
Swerve	2nd,2nd	3rd, 1st
Don't	1st, 3rd	4th, 4th

- In Battle of the Sexes you have the same top two

- In Chicken you don't.

Chicken

	Swerve	Don't
Swerve	0,0	-5, 10
Don't	10, -5	-10, -10

- Flipping a coin would still be good for both players.

Example (Chicken)
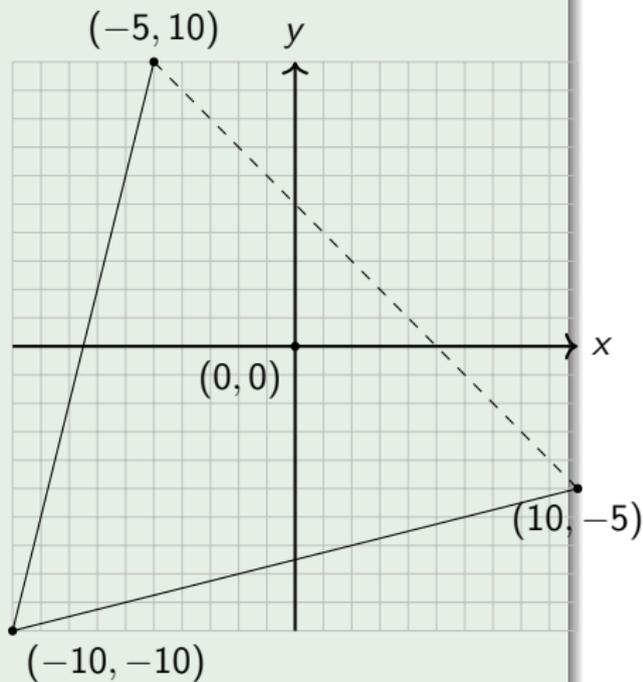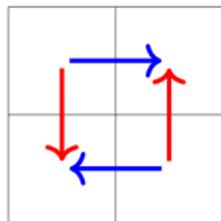
Chicken

Key Questions

- How to save face

- How to avoid blackmail or extortion

Example

- Any sort of dare

- Threats of war

- Threats of price wars

- Message bills with bad policy

- Tit-for-tat gerrymandering

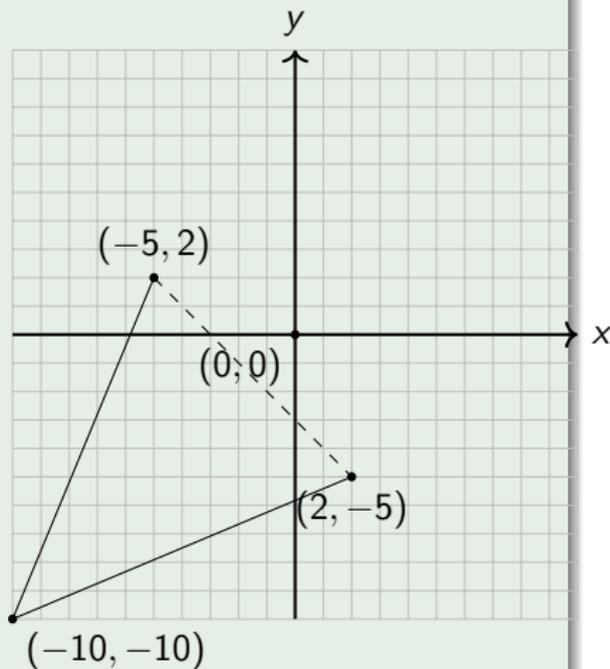|	Swerve	Don't
Swerve	0,0	-5, 2
Don't	2, -5	-10, -10

- Two saddle points
- Mixed Nash equilibrium: swerve $5/7$ of the time
    - That's more often than the $1/3$ we got last time
- Expected value is $-10/7$
    - That's *better* than before
    - Lowering payout improved the mixed Nash equilibrium!

Chicken

	Swerve	Don't
Swerve	0,0	-5, 2
Don't	2, -5	-10, -10

- This triangle isn't actually the payoff polygon

Example (Chicken Variant)

Chicken
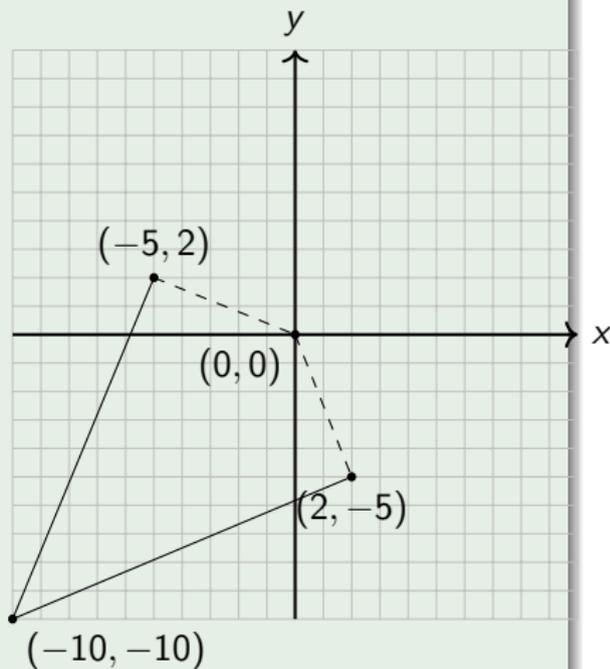
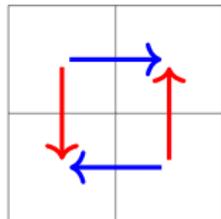|	Swerve	Don't
Swerve	0,0	-5, 2
Don't	2, -5	-10, -10

- This is the payoff polygon
- If they can coordinate, both players should just swerve.

Example (Chicken Variant)

Chicken Variations

	Swerve	Don't
Swerve	0,0	-5, 10
Don't	10, -5	-50, -50

- Two saddle points
- Mixed Nash equilibrium: swerve $9/11$ of the time
    - Most of the time!
- Expected value is $-10/11$
    - That's *better* than either other option
    - Even though the worst case is *really* bad.
- Because a crash is so bad, both parties are incentivized to swerve.

Chicken Variations

Example

- Mutually assured destruction
    - As war gets more destructive, people try harder to avoid it
    - Modernization has maybe moved war from the first variant to the second or third variant
- Congressional poison pills
    - Make a bill worse so it doesn't pass
- "Madman theory"
    - Act so crazy that people think you won't swerve anyway
- Cuban Missile Crisis

Prisoner's Dilemma

Example

- Two criminals are arrested and interrogated separately

- If neither confesses, they get off on a minor charge

- If both confess, both are convicted of a serious charge

- If *one* confesses, he gets off free but the other is convicted
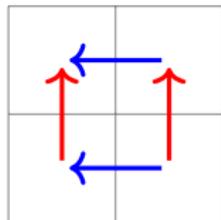 with no mercy

	Confess	Don't
Confess	-5,-5	0, -10
Don't	-10, 0	-2, -2
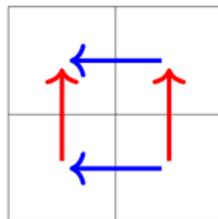
Discussion Question

- What should each player do?

Prisoner's Dilemma

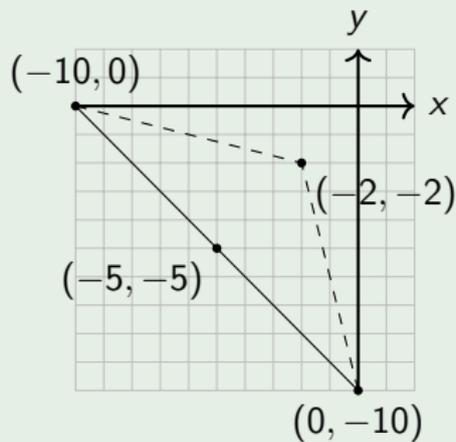|	Confess	Don't
Confess	-5,-5	0, -10
Don't	-10, 0	-2, -2

- Unique saddle point where both confess
- In fact, "Confess" dominates "Don't Confess"
- Simple game, right?

Prisoner's Dilemma

	Confess	Don't
Confess	-5,-5	0, -10
Don't	-10, 0	-2, -2

- Best outcome overall is neither confessing
- But that's *not* a Nash equilibrium.

Example (Prisoner's Dilemma)

Prisoner's Dilemma

- Maybe a good thing it's hard for criminals to coordinate?

- But these coordination problems show up everywhere.

	Confess	Don't
Confess	-5,-5	0, -10
Don't	-10, 0	-2, -2

	Defect	Cooperate
Defect	-5,-5	0, -10
Cooperate	-10, 0	-2, -2

Example

- Climate change
- Cleaning common spaces
- "Public goods"
- Arms Races

- Cartels
- Advertising
- Sports doping
- Large Cars

Prisoner's Dilemma

- The *two players* are better off cooperating

- May or may not be good for everyone else!

- Sometimes the public policy goal is to get people to cooperate

- Other times the goal is to prevent cooperation *against* society.

Penalize defectors

- Pass a law taxing pollution

- Sports leagues test for and ban doping

- Gangs can threaten snitches with violence

Subsidize cooperation

- Subsidies for clean energy

- Publicly funded cleanup efforts

Repeated Games

- A lot of our discussion has assumed players play once, without coordination
- In the real world that's not true
- I *know* if my roommate cleans the kitchen
- This opens up different types of strategies
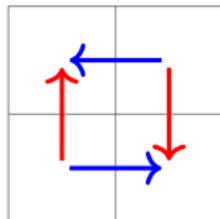
Tit for Tat

- Cooperate on turn 1
- Afterwards, play what your opponent did last time.
- Empirically does quite well

Stag Hunt

	Hare	Stag
Hare	5,5	8, 1
Stag	1, 8	10, 10

Discussion Question

- What should each player do?

	Hare	Stag
Hare	5,5	8, 1
Stag	1, 8	10, 10
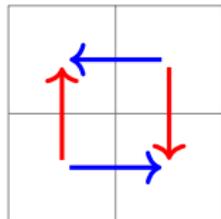
- Two saddle points
- Mixed Nash equilibrium plays Stag 2/3 of the time
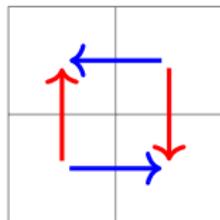 - Expected value is 7
  - Better than always-Hare, worse than always-Stag

Stag Hunt

	Hare	Stag
Hare	5,5	8, 1
Stag	1, 8	10, 10

Discussion Question

What is the prudent strategy?

- Hare has guarantee of 5

- Stag has guarantee of 1

- Hare is prudent.

- This is a problem of coordination

Stag Hunt versus Prisoner's Dilemma

	Defect	Coop
Defect	-5,-5	0, -10
Coop	-10, 0	-2, -2

	Hare	Stag
Hare	5,5	8, 1
Stag	1, 8	10, 10

	Defect	Coop
Defect	3rd, 3rd	1st, 4th
Coop	4th, 1st	2nd, 2nd

	Hare	Stag
Hare	3rd,3rd	2nd, 4th
Stag	4th, 2nd	1st, 1st

- In prisoner's dilemma, cooperation is unstable
- In a stag hunt, it's a stable equilibrium
  - (But if you expect your partner to defect, it's rational to defect as well.)