Gerrymandering

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Redistricting

- Apportionment: how many Congressional seats does each state get
- How does the *state* split up those seats?
- In the US: state k is divided into a_k districts, and each district elects a Congressperson
- How do we divide into districts?
- This process is called redistricting

Remark

There are other ways to approach this problem. We'll talk about them a little at the end. But in the US, this is pretty much always the problem we have to solve.

Redistricting

- After each census, each state draws a Congressional Map
- There are several typical constraints on how this map can be drawn:
  - Roughly equal population
    - Since *Reynolds v. Sims* in 1964
  - Contiguous
  - "relatively compact"
  - Minority representation
  - Respect natural geographic and political divisions
- Still a lot of maps that satisfy these conditions.

Gerrymandering

Definition

Gerrymandering is the act of drawing district boundaries for the purpose of benefiting a political group

- In the modern US, usually a political party
- Can be an ethnic group, economic class, or any other interest
- Frequently also used to protect specific incumbents

Packing and Cracking

- Gerrymandering typically involves two basic techniques.

Definition

- Packing involves putting many supporters of a party into a single district
- Cracking involves splitting the supporters of a party across many districts

- These sound like opposites
- Goal is to combine them cleverly to minimize representation of the opposing party.
- Concentrate support in a few districts they win easily
- Split the rest among a lot of districts they lose narrowly.

A Toy Example

- Taken from *Shape* by Jordan Ellenberg
- Imagine two parties, Orange and Purple
- 600,000 voters support Purple, and 400,000 support Orange
- Want to draw ten districts
- What is the fairest way to draw them?

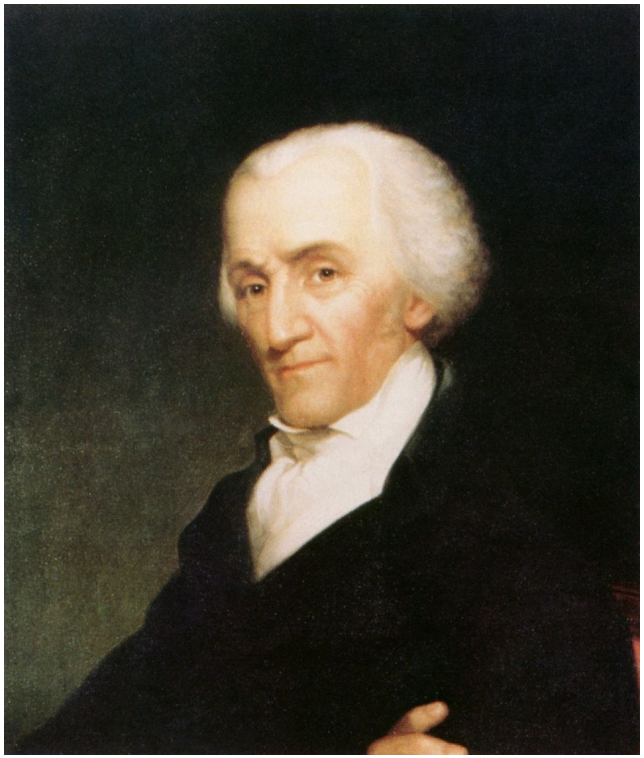
A Toy Example

	Option 1			Option 2		
	Purple	Orange		Purple	Orange	
District 1	75	25	P	45	55	O
District 2	75	25	P	45	55	O
District 3	75	25	P	45	55	O
District 4	75	25	P	45	55	O
District 5	75	25	P	45	55	O
District 6	75	25	P	45	55	O
District 7	35	65	O	85	15	P
District 8	35	65	O	85	15	P
District 9	40	60	O	80	20	P
District 10	40	60	O	80	20	P
	6	4		4	6	

A Toy Example

	Option 3			Option 4		
	Purple	Orange		Purple	Orange	
District 1	80	20	P	60	40	P
District 2	70	30	P	60	40	P
District 3	70	30	P	60	40	P
District 4	70	30	P	60	40	P
District 5	65	35	P	60	40	P
District 6	65	35	P	60	40	P
District 7	55	45	P	60	40	P
District 8	45	55	O	60	40	P
District 9	40	60	O	60	40	P
District 10	40	60	O	60	40	P
	7	3		10	0	

The Original Gerrymander

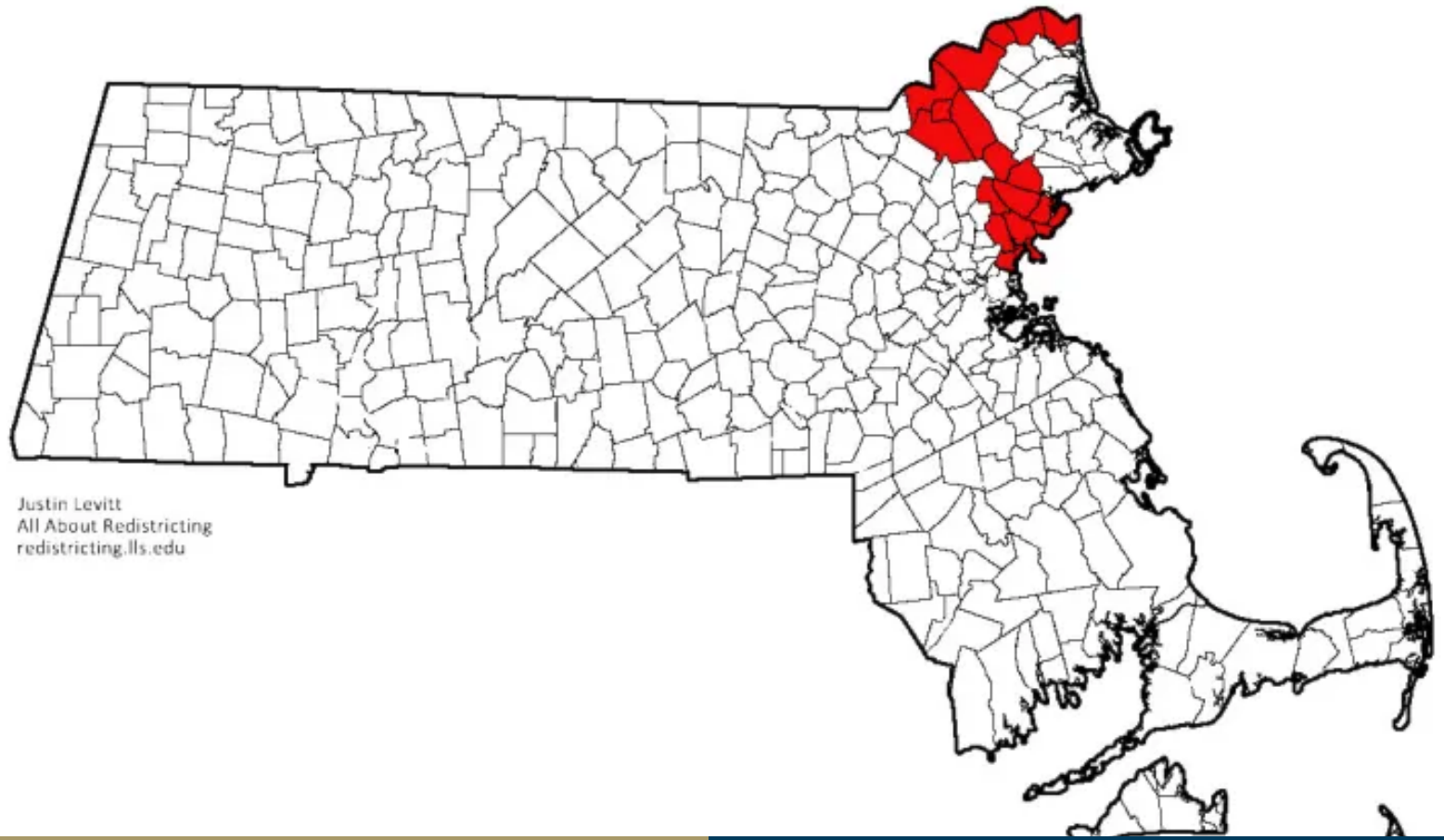


Founding Father Elbridge Gerry

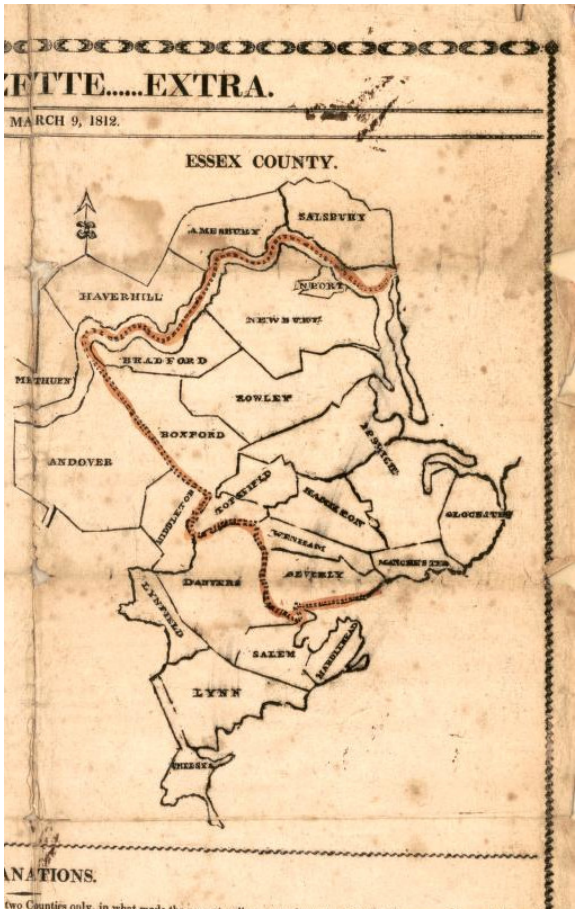
- Signed the Declaration of Independence
- Member of the Constitutional Convention
- Governor of Massachusetts 1810-1812
- Vice President 1813-1814
- Origin of the term Gerrymander

The Original Gerrymander

- Massachusetts 1812 districts for state senate
- Drawn to benefit the Democratic-Republican Party
- One particular district stood out

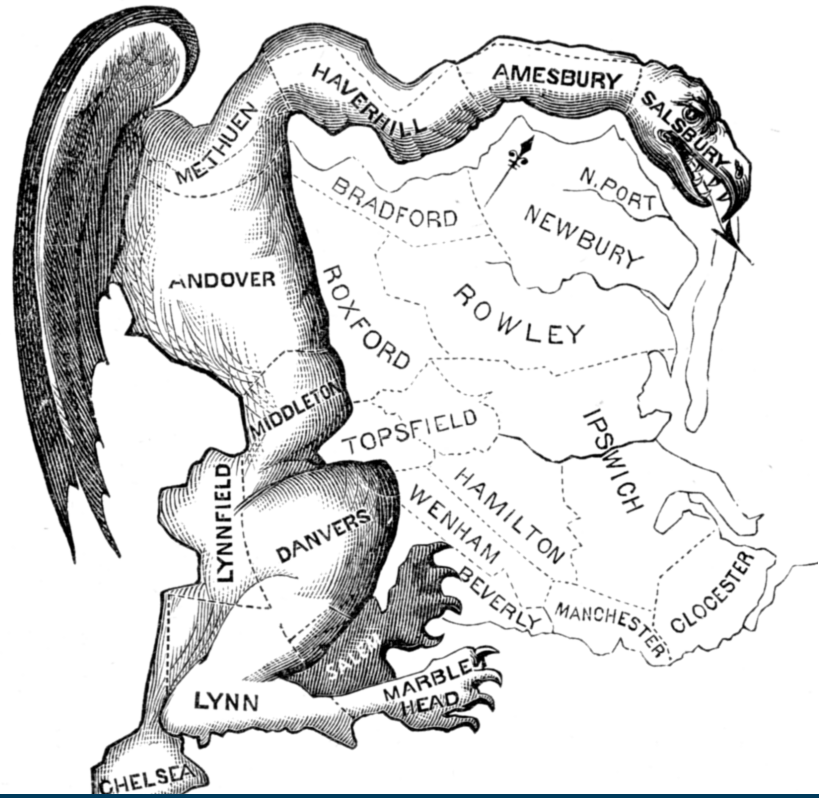


The Original Gerrymander



http://hdl.loc.gov/loc.gmd/g3763e.ct004086

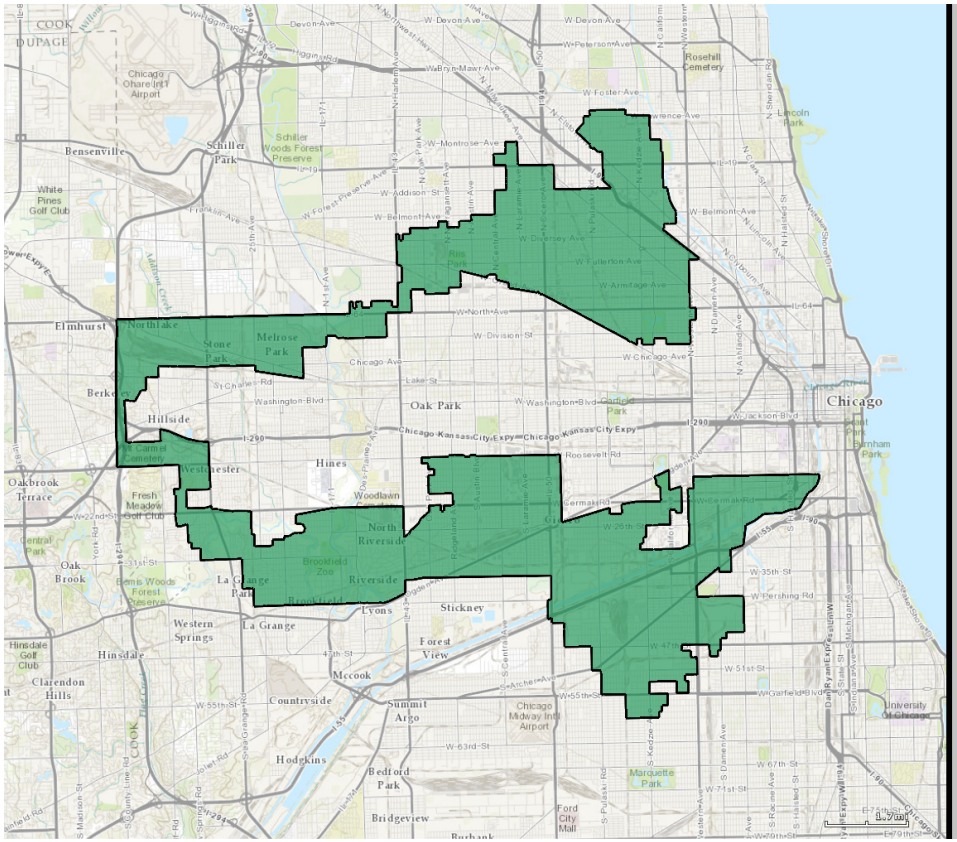
- Opponents claimed it looked like a lizard
- Political cartoon, probably by Elkanah Tisdale

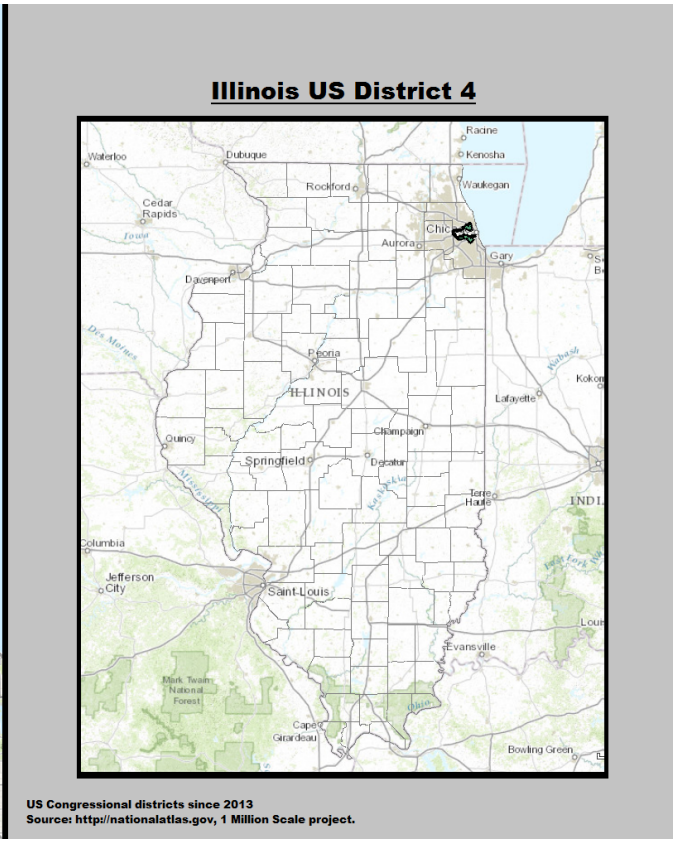


Modern Gerrymandering

- Gerrymandering has been around for centuries.
- Gotten much more intense recently Why?
  - Computers make gerrymandering easier and more precise
  - Better data also makes gerrymandering easier
  - Voters have gotten more consistent!
    - Packing and cracking assume you can identify "Republican voters" and "Democratic voters"
    - The more consistent voters are, the more productive this is.
- Original gerrymander looks pretty tame by modern standards.

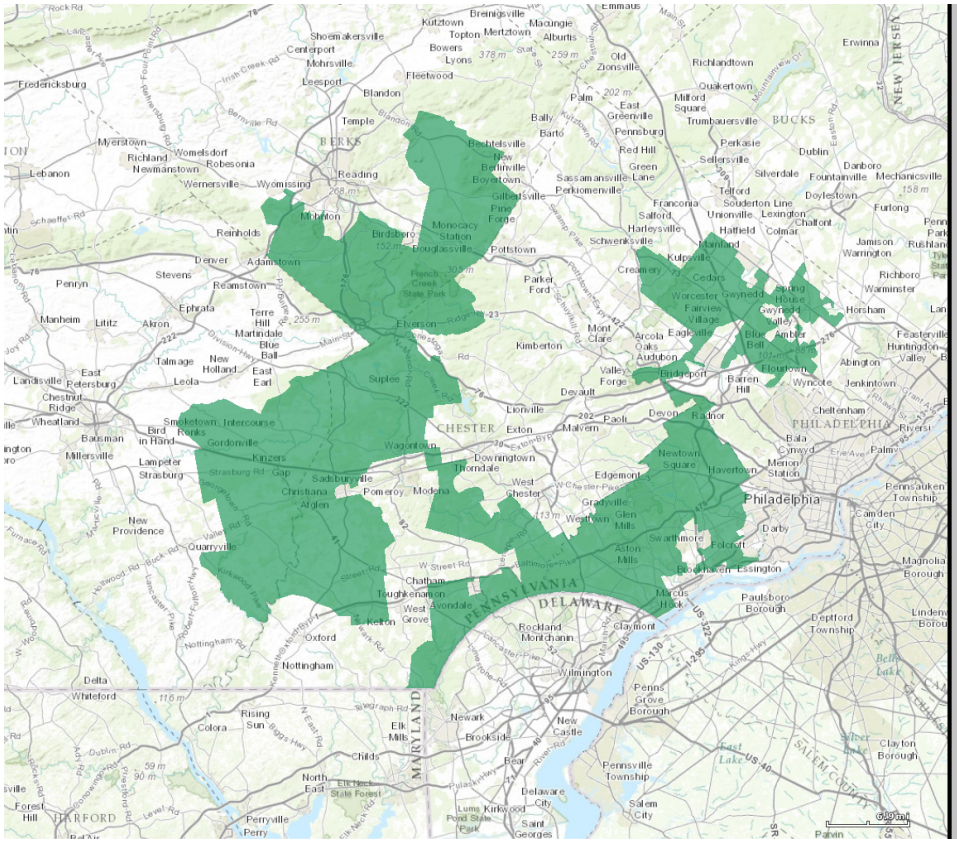
Illinois Congress District 4 in 2013



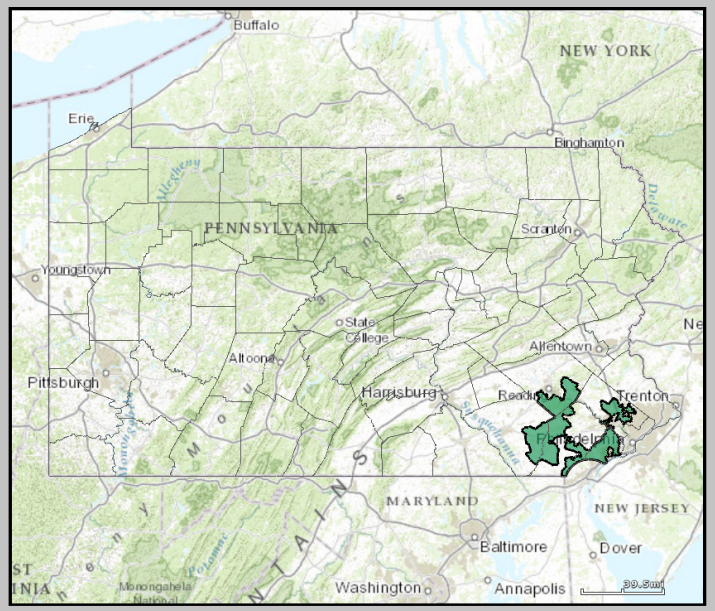


- "The Earmuffs"

Pennsylvania Congress District 7 in 2013



Pennsylvania US District 7



US Congressional districts since 2013
Source: http://nationalatlas.gov, 1 Million Scale project.

- "Goofy kicking Donald Duck"

Other Recent Developments

- Very effective gerrymander across the board in 2011
- Infamous Wisconsin gerrymander in 2011
- Many states develop mechanisms to prevent gerrymandering
  - Independent citizen redistricting commissions
  - State constitutional bans on partisan gerrymandering
- Supreme Court case *Rucho v. Common Cause* (2019)
  - North Carolina gerrymander by Republicans
  - Maryland gerrymander by Democrats
  - Court rules gerrymandering "nonjusticiable", no legal remedy
- Current effort to engage in mid-decade redistricting
  - Texas introducing aggressive Republican gerrymander
  - California retaliating with Democratic gerrymander

Compactness

- Idea: the problem is all these squiggly districts
- (The original Gerry-mander notable for its weird shape)
- Can we force the districts to be compact?
- How do we measure compactness?

Compactness: Polsby-Popper

- Compare area to perimeter
- First idea: Divide area by perimeter
- Problem: units don't work!
  - A 1×1 square would have $\dfrac{1 \times 1}{1+1+1+1} = 1/4$
  - A 2×2 square would have $\dfrac{2 \times 2}{2+2+2+2} = 1/2$
  - But they're the same shape, equally "compact"
- Fix: divide area by *square* of perimeter.

Definition

The Polsby-Popper score of a region is its area divided by its perimeter squared, $\dfrac{A}{p^2}$.

Compactness: Polsby-Popper

Example

- What is the Polsby-Popper score of a circle?
- A circle has area πr^2 and perimeter (circumference) $2\pi r$.
- The score is $\dfrac{\pi r^2}{(2\pi r)^2} = \dfrac{\pi r^2}{4\pi^2 r^2} = \dfrac{1}{4\pi} \approx 0.079.$

Example

- What is the Polsby-Popper score of a square?
- Has area s^2 and perimeter $4s$
- Score is $\dfrac{s^2}{(4s)^2} = \dfrac{s^2}{16s^2} = \frac{1}{16} = 0.0625$

Proposition

A circle has the largest possible Polsby-Popper score.

Compactness: Polsby-Popper

Example

- The Earmuffs have a score of about 0.005.

Example

- What is the Polsby-Popper score of a 1×100 rectangle?
- $$\frac{1 \times 100}{(1+1+100+100)^2} = \frac{100}{202^2} = \frac{25}{10201} \approx 0.0025.$$

Discussion Question

- Is a long thin rectangle a really awful region shape?
- Is it twice as bad as the earmuffs?

Compactness:Polsby-Popper

Discussion Question

- What is the Polsby-Popper score of Arizona's district 9?
- Why is this very hard to answer?

- Western border of Arizona is the Colorado River
- How long that is depends on how precisely we measure it
- Do we want to penalize geographic boundaries?

Compactness: the Convex Hull

- Maybe we don't care about boundaries that are *roughly* straight
- Want to penalize districts that have long extensions or obvious holes

Definition

A shape is convex if any line connecting two points in the shape is contained entirely within the shape.

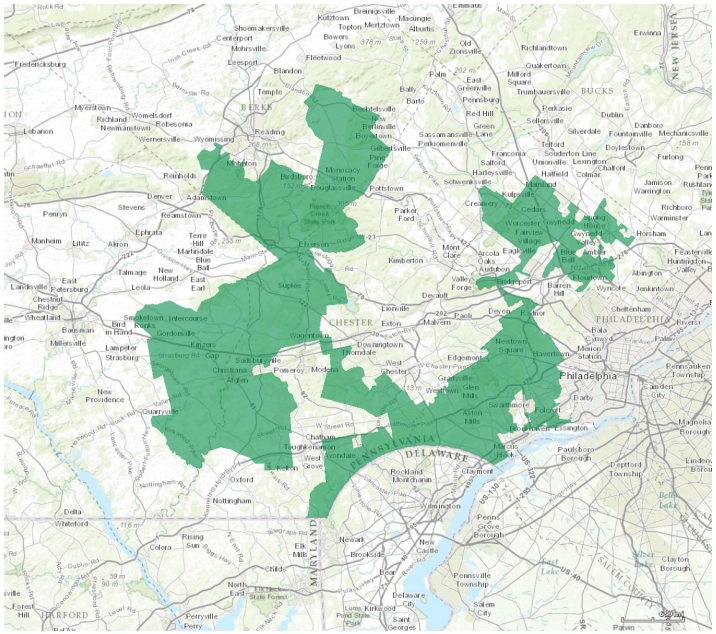
Example

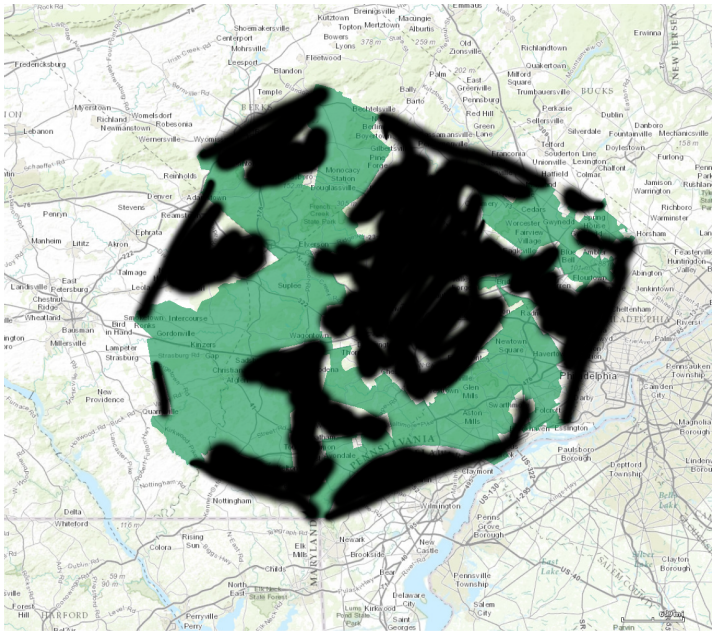
- Circles and rectangles are convex
- Stars and chevrons are not convex.
- The Gerry-mander, the Earmuffs, and Goofy Kicking Donald are not convex.

Compactness: the Convex Hull

Definition

- The convex hull of a shape is the smallest convex region containing the entire shape.
- The Population Polygon score of a district is the population of the district, divided by the population of its convex hull.

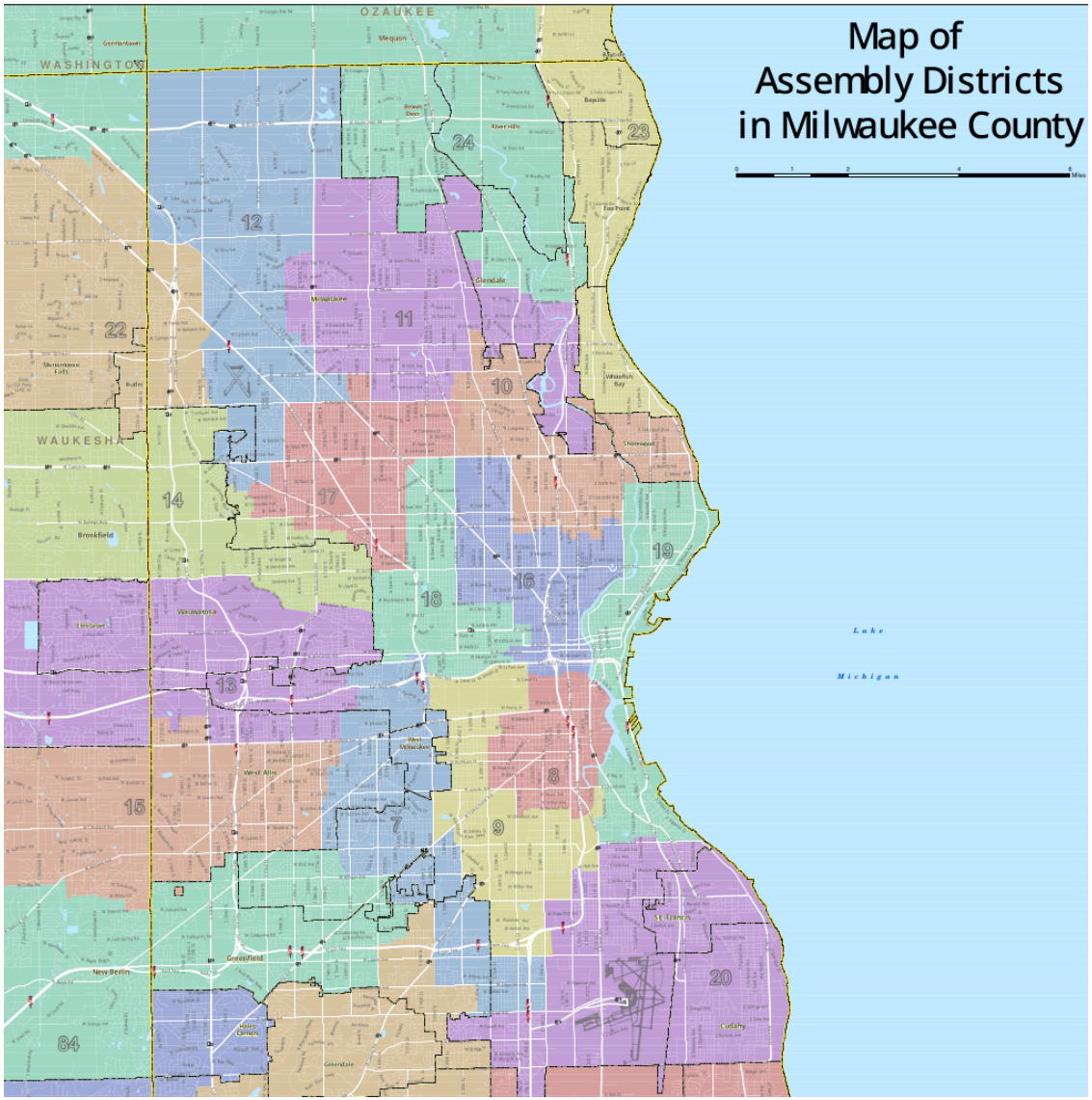




Compactness: the Convex Hull

- Population polygon is better than Polsby-Popper:
  - Considers population, not just geography
  - Doesn't penalize natural squiggly borders
- But it still doesn't *work*.

The Wisconsin Gerrymander



- This is the infamous gerrymander of the Wisconsin state assembly
- It *looks* fine!

Fairness

- Remember our four districting examples earlier:

	Purple	Orange
Actual Percentage	60%	40%
Option 1	6	4
Option 2	4	6
Option 3	7	3
Option 4	10	0

Discussion Question

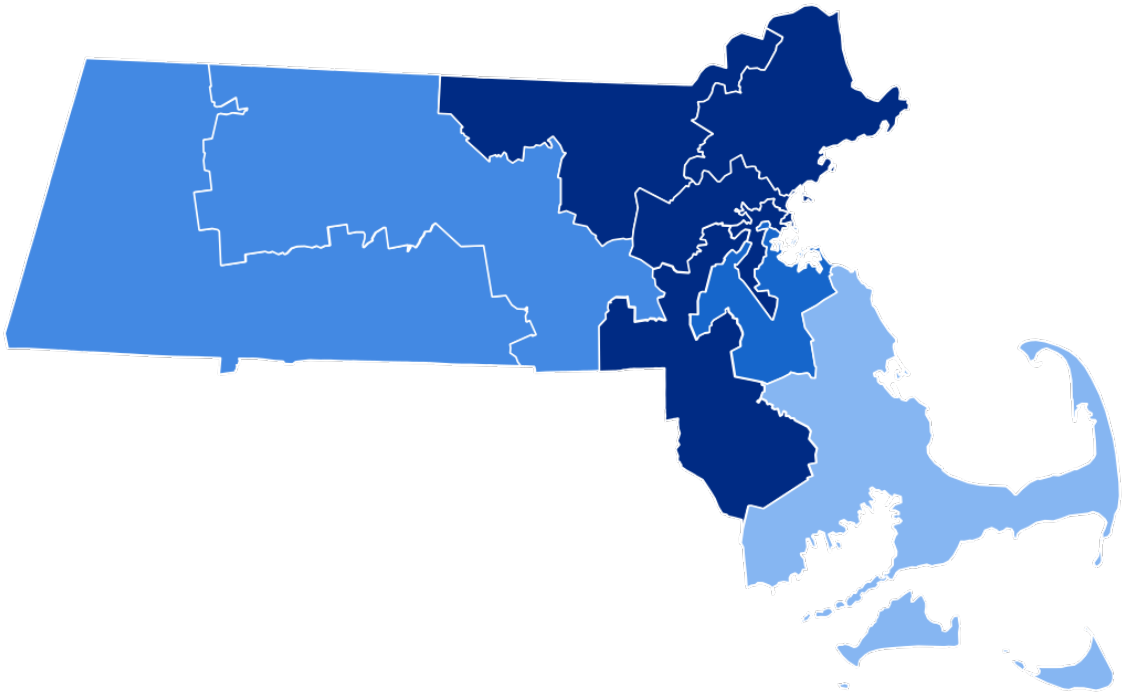
Which option is fairest? Which is least fair?

Fairness: Proportionality

- Idea: percentage of seats should match percentage of votes
- Many countries use proportional representation
- That would resolve this problem entirely, but the US has single-member districts
- Would a "fair map" produce proportional results?

Fairness: Proportionality

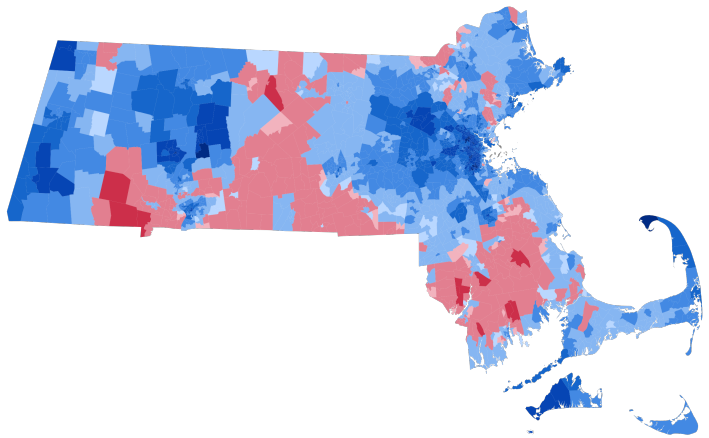
- Massachusetts has 9 Congressional seats
- In 2024, Kamala Harris got 63% of the vote
- 63% of the seats is 5.66, so proportional representation would give them 5 or 6 seats.
- They got nine.



Fairness: Proportionality

District	D %		District	D%
1st District	62.5		8th District	70.2
2nd District	68.6		9th District	56.4
All others	Uncontested			

- Massachusetts's Democrats are pretty evenly distributed
- Hard to produce Republican districts
- Maybe not possible to produce 3 Republican districts.



Efficiency and Wasted Votes

- A gerrymander wants a lot of narrow wins
- (and a few large losses)
- Winning by 5 is as good as winning by 50.

Definition

We say the wasted votes for a party are either

- votes cast for that party in a district where that party loses, or
- votes above 50% in a district where that party wins.

Definition

The efficiency gap between two parties is the difference in the numbers of wasted votes, divided by the total number of votes.

$$\text{Efficiency Gap} = \frac{\text{wasted votes for } A - \text{wasted votes for } B}{\text{total votes}}$$

Efficiency and Wasted Votes

Option 1			
Wasted	Purple	Orange	Wasted
25	75	25	25
25	75	25	25
25	75	25	25
25	75	25	25
25	75	25	25
25	75	25	25
35	35	65	15
35	35	65	15
40	40	60	10
40	40	60	10
300		200	

- Efficiency gap is $\frac{300 - 200}{1000} = 10\%/$

Efficiency and Wasted Votes

Option 2			
Wasted	Purple	Orange	Wasted
45	45	55	5
45	45	55	5
45	45	55	5
45	45	55	5
45	45	55	5
45	45	55	5
35	85	15	15
35	85	15	15
30	80	20	20
30	80	20	20
400		100	

- Efficiency gap is $\frac{400 - 100}{1000} = 30\%/$

Efficiency and Wasted Votes

- Encourage you to work out the efficiency gaps for Options 3 and 4
  - Option 3: Gap of zero
  - Option 4: Gap of 30%
- Is the option 3 gap surprising?
- Purple got 60% of the votes but 70% of the seats.
- Think back to Massachusetts—a 2 to 1 advantage led to a sweep

Efficiency and Wasted Votes

- With no efficiency gap, congress will be more lopsided than the popular vote
- We can even quantify that.

Proposition

If all districts have an equal number of votes, then the efficiency gap is

$$\Big(\textit{victory margin in vote}\Big) - \frac{1}{2}\Big(\textit{victory margin in seats}\Big)$$

- Efficiency gap says the result "should" be twice as dramatic as the voting percentage.
- Dangerously close to (skewed) proportional representation with more steps.

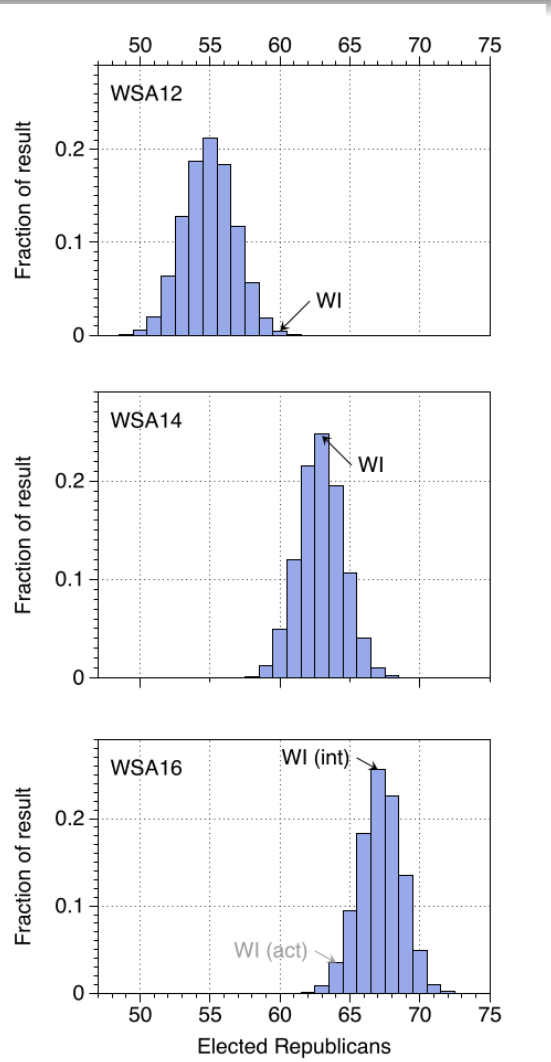
Random Ensembles

- Premise: want geographic representation, not proportional representation.
- The problem: can draw maps strategically.
- Can we have a computer draw them?
  - People viscerally hate this idea
  - But also not all maps are equally good
- Assign each map a score according to how well it satisfies traditional districting criteria.
- Idea: Pick the map with the highest score.
  - People still hate this idea
  - But also you can't do it. Too many options!

Random Ensembles

Definition

- Generate random maps, with a bias towards "nice" maps
- A collection of such maps is called an ensemble.



- Idea: Compute the result of election under each map in the ensemble
- Compare that to the result of the actual map.

Herschlag, Ravier, and Mattingly, Evaluating Partisan Gerrymandering in Wisconsin, 2017

Random Ensembles

How to generate random maps?

- Idea (Moon Duchin, Daryl DeFord, Justin Solomon): recombination
- Randomly choose two bordering districts
- Remove the border
- Randomly split combined district into two
- If new pair of districts is legally valid, keep it, otherwise reset
- Repeat a lot of times. (About a hundred thousand)

Random Ensembles

- Random ensembles don't eliminate gerrymandering
- They do let us measure it, sort of
- How unlikely would a map this extreme be, if we weren't trying to bias it?
- A lot of statistical tests work this way

Definition

- When running an experiment or survey, often compute the *p*-value
  - Choose a default assumption, the null hypothesis
  - Run your experiment and measure result
  - The *p*-value is the probability of getting a result at least that extreme, if the null hypothesis is true
- Often say a result is statistically significant if $p < 0.05$.

Final Exam Info

Test Rules

- Monday December 15, 12:40-2:40 PM
- In this room
- Expect about ten questions
- Bring a two-sided handwritten note sheet
- You can bring a calculator, and should

Final Exam Info

Test Topics

- Exam is comprehensive
- 1-2 questions on voting
- 1-2 questions on apportionment
- Remaining questions on game theory
  - Naive and prudent strategies
  - min-max diagrams, flow diagrams, saddle points
  - Probability and expected value
  - Mixed strategies
  - Neutralizing strategies, Nash Equilibria
  - Solving 2×2 games
  - Non-zero sum game flow diagrams and saddle points
  - Familiarity with five important games we discussed

Thanks for a great class!