# **Evaluating Voting Methods I**

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# Some Theorems

## Proposition

Any social choice function that satisfies anonymity and neutrality must violate decisiveness.

# Proposition (Taylor)

No social choice function involving at least three candidates satisfies both independence and the Condorcet criterion.

# Proposition

If a method is Condorcet then it satisfies the majority criterion.

#### Proof.

- Suppose A has a majority of first-place votes.
- Then A will win any head-to-head matchup, so A is the Condorcet candidate.
- Any method that satisfies the Condorcet criterion will cause A to win, also satisfying the majority criterion.

#### Remark

We can say the Condorcet criterion is "stronger" than the majority criterion.

# Summary

# Voting Methods

Plurality

- Hare's method
- Coombs's Method

dictatorship

monarchy

all ties

- Copeland's Method
- Borda count
- More to come!

## Voting Method Criteria

- unanimous
- decisive

majoritarian

- anonymous
- neutral

monotone

Pareto

- independent
- Condorcet
- anti-Condorcet

# Analyzing the Plurality Method

## Proposition

The plurality method is majoritarian, monotone, and Pareto, but not Condorcet, anti-Condorcet, or independent.

- The majority is always a plurality, so candidate with majority will win.
- Monotone: raising a candidate on some preference lists can't reduce their first-place votes, or increase anyone else's
- Pareto: if A is ahead of B on every preference list, then B gets no votes, and can't win.

# Analyzing the Plurality Method

#### Claim

The plurality method is neither Condorcet not anti-Condorcet.

2	3	2

Α	В	C

- B wins plurality, but
- A beats B, A beats C, and C beats B.
- A is Condorcet, and loses
- B is anti-Condorcet, and wins

# Analyzing the Plurality Method

## Claim

The plurality method is not independent.

Α	А	Α	Α	В	В	В		А	Α	С	С	В	В	В
В	В	С	С	А	Α	Α	$\rightarrow$	В	В	Α	Α	Α	Α	А
С	С	В	В	С	С	С		С	С	В	В	С	С	С

- First profile: A wins, B and C lose
- Second profile: B wins, A and C lose
- But relative position of A and B doesn't change.

#### Definition

The *antiplurality method* names as winner the candidate with the fewest last-place votes.

## Example

5	4	4	4	3
Α	В	С	D	Е
В	С	В	В	D
С	Е	D	Е	В
Е	D	Е	С	С
D	A	A	A	A

B, C, E all win.

5	4	4	4	3
В	С	Α	D	Е
С	Α	В	А	Α
Е	В	Е	В	В
D	Е	D	Е	D
Α	D	С	С	С

B and E both win.

#### Definition

The *antiplurality method* names as winner the candidate with the fewest last-place votes.

### Poll Question

Which criteria does the antiplurality method satisfy?

## Proposition

The antiplurality method is monotone, but not majoritarian, Condorcet, anti-Condorcet, Pareto, or independent.

## Proposition

The antiplurality method is monotone, but not majoritarian, Condorcet, anti-Condorcet, Pareto, or independent.

#### Claim

The antiplurality method is monotone.

- Raising a candidate in preference lists:
  - Can't increase their last-place votes
  - Can't decrease anyone else's last-place votes.
- If a candidate wins before getting raised, they win after.



#### Claim

The antiplurality method is not majoritarian, Condorcet, or anti-Condorcet.

#### Proof.

Consider the profile:

00 u p. 0				
С	С	В	В	В
Α	Α	С	Α	Α
В	В	Α	С	С

- What happens? A wins.
- B gets a majority but loses. Not majoritarian.
- B is the Condorcet winner. Not Condorcet.
- A is anti-Condorcet candidate.
  Not anti-Condorcet.

## Claim

The antiplurality method is not Pareto.

- This is surprising!
- Just barely true. But true.

## Proof.

Consider the profile:

Α	Α	А
В	В	В
С	С	С

- What happens? A and B both win.
- Every voter prefers A to B, but B wins.
- Not Pareto.



## Claim

The antiplurality method is not independent.

#### Proof.

Consider:

С	Α	Α	В	В
Α	В	В	Α	Α
В	С	С	С	С

С	Α	Α	В	В
Α	В	В	С	С
В	С	С	Α	Α

- What happens?
  - A wins profile 1
  - B wins profile 2
- No voter has changed preferences between A and B.
- Not independent.

#### Definition

Eliminate the candidate(s) who have the fewest first-place votes. Repeat. The last remaining candidate(s) are the winner(s).

- Popular and widely used
- Australia, New Guinea, Alaska, Maine, NYC
- Surprisingly bad at our criteria.

## Proposition

Hare's method is majoritarian and Pareto, but not monotone, Condorcet, anti-Condorcet, or independent.

#### Claim

Hare's method is majoritarian.

#### Proof.

- A majority of first-place votes will never be the fewest
- They will never be eliminated, and everyone else will.

### Claim

Hare's method is Pareto.

- If everyone prefers A to B, then B has no first-place votes
- B gets eliminated in the first round, and can't win.

## Claim

Hare's method is not monotone.

6	5	4	2	
Α	С	В	В	_
В	Α	С	Α	
С	В	Α	С	

6	5	4	2
А	С	В	Α
В	Α	С	В
С	В	Α	С

- Profile 1: Eliminate C, then B. A wins.
- Profile 2: Eliminate B, then A. C wins.
- Getting more votes makes A lose.

#### Claim

Hare's method is not Condorcet or anti-Condorcet.

## Proof.

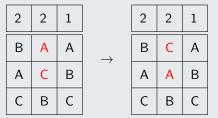
### Consider:

2	3	2
Α	В	С
С	Α	Α
В	C	В

- What happens?
  - Eliminate A and C; B wins.
- B is the anti-Condorcet candidate and wins
- A is the Condorcet candidate and loses.

## Claim

Hare's method is not independent.



- Profile 1: eliminate C, then B. A wins.
- Profile 2: eliminate A, then C. B wins.
- Relative preference of A and B has not changed