

Evaluating Voting Methods III

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Summary

Voting Methods

- Plurality
- Hare's method
- Coombs's Method
- dictatorship
- monarchy
- all ties
- Copeland's Method
- Borda count
- Antiplurality

Voting Method Criteria

- unanimous
- decisive
- majoritarian
- anonymous
- neutral
- monotone
- Pareto
- independent
- Condorcet
- anti-Condorcet

Previous Results

Proposition

The plurality method is majoritarian, monotone, and Pareto, but not Condorcet, anti-Condorcet, or independent.

Proposition

The antiplurality method is monotone, but not majoritarian, Condorcet, anti-Condorcet, Pareto, or independent.

Proposition

Hare's method is majoritarian and Pareto, but not monotone, Condorcet, anti-Condorcet, or independent.

Previous Results

Proposition

Coombs's method is Pareto, but not majoritarian, monotone, Condorcet, anti-Condorcet, or independent.

Proposition

The Borda count method is monotone, anti-Condorcet, and Pareto, but not majoritarian, Condorcet, or independent.

Proposition

Copeland's method is majoritarian, Condorcet, anti-Condorcet, monotone, and Pareto.

Discussion Question

Is Copeland's method independent?

Copeland's method

Claim

Copeland's method is monotone.

Proof.

- Moving A up on some lists won't hurt them in any head-to-head, so won't reduce A's score
- Won't affect any other head-to-head at all
- So it can't increase any other candidate's score
- If A wins before the switch, will also win after.



Discussion Question

How does this suggest we look at independence?

Copeland's method

Claim

Copeland's method is not independent.

Proof.

B	A	A
C	B	C
A	C	B

→

B	A	C
C	B	A
A	C	B

- Profile 1: A gets 2, B gets 1, C gets 0. A wins.
- Profile 2: A gets 1, B gets 1, C gets 1. All candidates win
- A and B haven't changed relative positions
- B loses in profile 1 and wins in profile 2



Proposition

Copeland's method is majoritarian, Condorcet, anti-Condorcet, monotone, and Pareto, but not independent.

Black's method

- Duncan Black (1908-1991)
- *The Theory of Committees and Elections* (1958)

Definition

Black's method is the social choice function that chooses the Condorcet candidate as the unique winner if there is a Condorcet candidate, and chooses the Borda count winner if there is not.

- Attempt to combine benefits of multiple methods
- Start with Borda count, “fix” the “problem” that it’s not Condorcet.

Discussion Question

What criteria will Black's method satisfy?

Black's method

Claim

Black's method is Condorcet and majoritarian.

Proof.

- Condorcet by definition.
- Majoritarian because Condorcet.



Claim

Black's method is anti-Condorcet

Proof.

- anti-Condorcet candidate isn't Condorcet
- anti-Condorcet candidate can't win Borda count



Black's Method

Claim

Black's method is monotone.

Proof.

- Two cases
- If A is Condorcet winner, moving them up in some rankings won't change that, so they still win.
- If no Condorcet winner and A wins by Borda count:
 - Moving A up can't lower their score or raise anyone else's score, so they win Borda count
 - Moving A up can't make anyone *else* into Condorcet winner
 - Moving A up could make A the Condorcet winner, but that's fine.



Black's Method

Claim

Black's method is Pareto.

Proof.

- Suppose all voters prefer A to B
- B is not the Condorcet candidate since they lose to A
- B can't win the Borda count because A will have more points
- B can't win.



Black's Method

Claim

Black's method is not independent.

Proof.

- Easy answer: it's Condorcet, so not independent. Or:

B	A	A
C	B	C
A	C	B

→

B	A	C
C	B	A
A	C	B

- Profile 1: A is the Condorcet candidate, and wins
- Profile 2: no Condorcet candidate
 - Each candidate gets 3 Borda points
 - All three are winners

Proposition

Black's method is majoritarian, Condorcet, anti-Condorcet, monotone, and Pareto, but not independent.

Poll Question

- Which properties have been common?
- Which have been uncommon?

Discussion Question

- Why is independence so uncommon?
- Do any methods achieve independence?

Dictatorship

Definition

One voter is the dictator. Their first choice is the unique winner.

Discussion Question

What criteria does the dictatorship method satisfy?

Dictatorship

Claim

The dictatorship method is monotone.

Proof.

- If A wins, they're at the top of the dictator's preference list
- Moving them up on other lists won't change that.



Dictatorship

Claim

The dictatorship method is Pareto.

Proof.

- If A is higher than B on every list, then A is higher on the dictator's list
- B isn't at the top of the dictator's list, and can't win.



Dictatorship

Claim

The dictatorship method is independent.

Proof.

- Suppose A wins and B loses in profile 1.
- Then A is at the top of the dictator's preference list in profile 1.
- If profile 2 has A and B in the same relative positions, then B is not at the top of the dictator's preference list in profile 2.
- B can't win in profile 2.



Dictatorship

Claim

The dictatorship method is not Condorcet, anti-Condorcet, or majoritarian.

Proof.

Consider:

↓

A	A	B
B	B	A

- What happens?
- B wins
- A is the Condorcet candidate
- A is the majority candidate
- B is the anti-Condorcet candidate.



Proposition

The dictatorship method is monotone, Pareto, and independent, but not Condorcet, anti-Condorcet, or majoritarian.

Constant Functions

Proposition

The all-ties method and the monarchy method are monotone and independent, but not Condorcet, anti-Condorcet, majority, or Pareto.

Proof.

- **Constant functions:** same output for any input
- Monotone and independent, because no candidate can win in one profile but lose in another
- Violate Condorcet, anti-Condorcet, majority, and Pareto, because rankings have no effect on who wins.



Summary

	anon	neu	unan	dec	maj	Con	AC	mono	Par	ind
Plurality	Y	Y	Y	N	Y	N	N	Y	Y	N
Antiplur	Y	Y	Y	N	N	N	N	Y	N	N
Borda	Y	Y	Y	N	N	N	Y	Y	Y	N
Hare	Y	Y	Y	N	Y	N	N	N	Y	N
Coombs	Y	Y	Y	N	N	N	N	N	Y	N
Copeland	Y	Y	Y	N	Y	Y	Y	Y	Y	N
Black	Y	Y	Y	N	Y	Y	Y	Y	Y	N
Dictator	N	Y	Y	Y	N	N	N	Y	Y	Y
All-ties	Y	Y	N	N	N	N	N	Y	N	Y
Monarchy	Y	N	N	Y	N	N	N	Y	N	Y

Proposition (Taylor)

No social choice function involving at least three candidates satisfies both independence and the Condorcet criterion.

- Condorcet isn't *that* common

Discussion Question

Why is independence so hard?

The Condorcet Paradox

Example

A	C	B
B	A	C
C	B	A

- Who should win?

Definition

- A **Condorcet Paradox** occurs when every candidate loses to at least one other candidate in a simple-majority head-to-head matchup.
- In this case we will get a cycle of candidates such that each beats the next head-to-head, until the last beats the first.

The Condorcet Paradox

Example

146	145	144
D	C	B
E	D	C
F	E	D
G	F	E
A	G	F
B	A	G
C	B	A

- Suppose we start with policy D
- Propose switch to C
- Passes with 289 votes
- Switch to B passes with 290 votes
- Switch to A passes with 291 votes
- Everyone is less happy!