

**Problem 1.** (a) Suppose we have a monotone voting method that makes  $C$  win in the first profile below. What can you say about the result in the second profile, and why?

|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| 3 | 2 | 2 |   | 3 | 2 | 2 |
| B | A | C |   | C | A | C |
| C | C | B | → | B | C | B |
| A | B | A |   | A | B | A |

**Solution:**  $C$  must win in the second profile, because they have moved up in rank in some preference lists and so by monotone they must win.

(b) Suppose we have a Pareto voting method. What can you say about the result in the profile below, and why?

|   |   |   |   |
|---|---|---|---|
| 2 | 5 | 4 | 3 |
| A | B | A | B |
| D | A | D | A |
| C | C | B | D |
| B | D | C | C |

**Solution:**  $D$  must lose in this profile, since every voter prefers  $A$  to  $D$  and the method is Pareto.

**Problem 2.** (a) Explain why Copeland's method is Condorcet.

**Solution:** Suppose  $A$  is a Condorcet candidate. Then they will win every matchup, and thus get the maximum number of points.

No other candidate will win every matchup, so none of them will get that many points. So  $A$  will be the unique winner of the election.

Maybe a slightly better more precise version: Suppose we have an election with  $n$  candidates, and  $A$  is a Condorcet candidate. Then  $A$  will win every matchup, and thus get  $n - 1$  points. Every other candidate will lose to  $A$ , and thus will get at most  $n - 2$  points. Thus  $A$  has the most points and will be the unique winner.

- (b) Give an example showing that the Borda count is not independent. (Explain why it shows this!)

**Solution:** There are many correct answers but one of them is this:

|   |   |   |   |   |
|---|---|---|---|---|
| 2 | 3 |   | 2 | 3 |
| A | B |   | A | B |
| C | A | → | B | A |
| B | C |   | C | C |

In profile 1,  $A$  gets  $4 + 3 = 7$  points,  $B$  gets  $0 + 6 = 6$  points, and  $C$  gets  $2 + 0 = 2$  points, so  $A$  wins and  $B$  and  $C$  lose.

In profile 2,  $A$  gets 7 points but  $B$  gets 8 points and  $C$  gets 0. This makes  $B$  the unique winner.

So in profile 1,  $A$  wins and  $B$  loses. In profile 2,  $B$  wins, even though the relative rankings of  $A$  and  $B$  have not changed.