Math 1231: Single-Variable Calculus 1

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Recitation 1

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Review Questions

These are all questions on material you should have learned in pre-calculus, but that you might be rusty on. Give them a try and talk to each other! But don't use a calculator; that will get in the way of what you should be learning.

(a) What are the solutions to $x^2 + 5x + 6 = 0$?

(b) What are the solutions to $x^2 + 5x + 5 = 0$?

(c) Compute $(3x + 5)^2$.

(d) Factor $x^3 - 8$.

(e) Compute $\sin(\pi/3)$.

(f) Compute $\tan(5\pi/6)$.

(g) Compute $\sec(-\pi/4)$.

Estimation

The first thing we're going to discuss in class is the idea of *estimation*. Most of the time we compute things exactly: $3 \times 5 = 15$. Other times we estimate: $32 \times 49 \approx 1500$, but this isn't exactly true.

But I want to raise a new question, in between these two. We want to estimate things, but we want to estimate them *well*, and be able to say exactly how well we're estimating them.

Problem 1. For example, maybe we want to multiply two numbers together and get 1500 ± 25.

(a) The notation 1500 ± 25 means within 25 units of 1500. What is the largest number that satisfies this? What is the smallest number?

(b) Write your answer to part (a) in interval notation.

(c) Is 32×49 inside this interval?

(d) Give two numbers x and y so that $x \cdot y$ is in the 1500 ± 25 interval, but is *not* exactly equal to 1500.

We want to think of this in terms of functions (as we think of most things in college math courses). So we have some rule with an input and an output, and we want to get the output really close to some target output. This often makes sense in engineering contexts, when we're trying to build something with specific properties.

Problem 2. A cylindrical water tank has a base with an area of six square inches. We want to fill it with water ten inches deep, to the nearest inch.

(a) What is the exact volume of water would we ideally want to pour in?

(b) What is the least height that would satisfy our requirements? What is the least volume of water we can pour into the tank and still satisfy our height requirements?

(c) What is the greatest height that would satisfy our requirements? What is the greatest volume of water we can pour into the tank and satisfy our height requirements?

(d) How much error can we have in the volume? That is, how far away from our ideal volume in part (a) can we get and still be safe?

Problem 3. We know that $\sqrt{4} = 2$. Suppose we want to have $1 \leq \sqrt{x} \leq 3$. We might describe this as wanting $\sqrt{x} = 2 \pm 1$. If we're being fancy, we use the Greek letter ε and say that we want an error tolerance of $\varepsilon = 1$.

(a) What is the largest input that keeps the output within ε of 2?

(b) What is the smallest input that keeps the output within ε of 2?

(c) Our ideal input is 4. How much can we miss our ideal input by? (When we're being fancy, we call this number by the Greek letter δ).

Now instead let's take $\varepsilon = .5$.

(d) What is the largest input that keeps the output within ε of 2?

(e) What is the smallest input that keeps the output within ε of 2?

(f) What does that make δ?