Math 1231: Single-Variable Calculus 1
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Recitation 1

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Review Questions

These are all questions on material you should have learned in pre-calculus, but that you might be rusty on. Give them a try and talk to each other! But don't use a calculator; that will get in the way of what you should be learning.

(a) What are the solutions to $x^2 + 5x + 6 = 0$?

Solution: We have $x^2 + 5x + 6 = (x + 2)(x + 3)$, so the solutions are $x = -2$ and $x = -3$.

(b) What are the solutions to $x^2 + 5x + 5 = 0$?

Solution: We can't really factor this, so instead we use the quadratic formula. So the solutions are

$$x = \frac{-5 \pm \sqrt{25 - 20}}{2} = \frac{-5}{2} \pm \frac{\sqrt{5}2}{.}$$

(c) Compute $(3x + 5)^2$.

Solution: You can FOIL this, or know the formula for squaring a polynomial. either way the answer is $9x^2 + 30x + 25$.

Importantly the answer is *not* $\xcancel{9x^2 + 25}$.

(d) Factor $x^3 - 8$.

Solution: This relies on a "difference of cubes" formula. There is a rule that $a^3 - b^3 = (a-b)(a^2+ab+b^2)$, so $x^3 - 8 = (x-2)(x^2+2x+4)$.

(e) Compute $\sin(\pi/3)$.

Solution: $\sin(\pi/3) = \sqrt{3}/2$.

(f) Compute $\tan(5\pi/6)$.

Solution: $\sin(5\pi/6) = 1/2$ and $\cos(5\pi/6) = -\sqrt{3}/2$ so $\tan(5\pi/6) = \frac{1/2}{-\sqrt{3}/2} = \frac{1}{-\sqrt{3}}$. You may want to clean that answer up to $\frac{-1}{\sqrt{3}}$ or even $\frac{\sqrt{3}}{3}$ but the first version is fine.

(g) Compute $\sec(-\pi/4)$.

Solution: $\cos(-\pi/4) = \sqrt{2}/2$ so $\sec(-\pi/4) = \frac{2}{\sqrt{2}}$. You could also clean this up and just say $\sec(-\pi/4) = \sqrt{2}$.

Estimation

The first thing we're going to discuss in class is the idea of *estimation*. Most of the time we compute things exactly: $3 \times 5 = 15$. Other times we estimate: $32 \times 49 \approx 1500$, but this isn't exactly true.

But I want to raise a new question, in between these two. We want to estimate things, but we want to estimate them *well*, and be able to say exactly how well we're estimating them.

Problem 1. For example, maybe we want to multiply two numbers together and get 1500 ± 25.

(a) The notation 1500 ± 25 means within 25 units of 1500. What is the largest number that satisfies this? What is the smallest number?

(b) Write your answer to part (a) in interval notation.

(c) Is 32×49 inside this interval?

(d) Give two numbers x and y so that $x \cdot y$ is in the 1500 ± 25 interval, but is *not* exactly equal to 1500.

Solution:

(a) The largest number is $1500 + 25 = 1525$ and the smallest is $1500 - 25 = 1475$.

(b) The interval is $[1475, 1525]$.

There's actually kind of an interesting interpretive question here. In math courses, our arguments are easier to write with "strict" inequalities, so we want to consider the open interval $(1475, 1525)$. However, in practical applications people tend to find it easier to work with inclusive intervals. Either is a reasonable way to approach the topic!

(c) $32 \times 49 = 1568$ is not in the interval, so that's not close enough.

(d) There are very many valid answers. But the simplest is probably $31 \times 49 = 1519$ is in the interval.

$32 \times 48 = 1536$ is not in the interval, but $32 \times 47 = 1504$ definitely is.

We want to think of this in terms of functions (as we think of most things in college math courses). So we have some rule with an input and an output, and we want to get the output really close to some target output. This often makes sense in engineering contexts, when we're trying to build something with specific properties.

Problem 2. A cylindrical water tank has a base with an area of six square inches. We want to fill it with water ten inches deep, to the nearest inch.

(a) What is the exact volume of water would we ideally want to pour in?

(b) What is the least height that would satisfy our requirements? What is the least volume of water we can pour into the tank and still satisfy our height requirements?

(c) What is the greatest height that would satisfy our requirements? What is the greatest volume of water we can pour into the tank and satisfy our height requirements?

(d) How much error can we have in the volume? That is, how far away from our ideal volume in part (a) can we get and still be safe?

Solution:

(a) 60 cubic inches.

(b) The least height that would satisfy our requirements is 9.5 inches. (This is a bit of a trick question; if we're rounding we need 9.5 or more, and 9 won't work.) The volume that would give us this height is 57 cubic inches.

(c) The greatest height that would work is 10.5 inches, and the volume that would give this height is 63 cubic inches.

(d) The error in our volume can be at most 3 cubic inches: $63 - 60 = 3$ and also $60 - 57 = 3$.

Problem 3. We know that $\sqrt{4} = 2$. Suppose we want to have $1 \leq \sqrt{x} \leq 3$. We might describe this as wanting $\sqrt{x} = 2 \pm 1$. If we're being fancy, we use the Greek letter ε and say that we want an error tolerance of $\varepsilon = 1$.

(a) What is the largest input that keeps the output within ε of 2?

(b) What is the smallest input that keeps the output within ε of 2?

(c) Our ideal input is 4. How much can we miss our ideal input by? (When we're being fancy, we call this number by the Greek letter δ).

Now instead let's take $\varepsilon = .5$.

(d) What is the largest input that keeps the output within ε of 2?

(e) What is the smallest input that keeps the output within ε of 2?

(f) What does that make δ?

Solution:

(a) 9

(b) 1

(c) This is actually kind of a trick question. You *could* say that the inputs need to be 5 ± 4 so $\delta = 4$, but that's not how we want to think about it in this course. (Anyone who came up with that answer isn't really wrong; they're just answering a slightly different question.)

Since the input that gives us the *exact* answer we want is 4, we're looking for $4 \pm \delta$. We can overshoot our ideal by 5, but we can only undershoot it by 3. So if we don't know which direction we're going to miss in, we're only safe if we miss by 3 or less. So we set $\delta = 3$.

(You could imagine, in a real-world process, choosing to aim for 5. You'd accept being too high on average in exchange for it being easier to stay within the error margin overall. But that's not how we generally want to approach this topic.)

(d) 6.25

(e) 2.25

(f) On the low end, we can miss by 1.75; on the high end we can miss by 2.25. So we take $\delta = 1.75$. (Again, you "could" take 4.25 ± 2, but that's answering a slightly different question.)